Quantum information and control in open systems: adiabatic approaches

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1 Adiabatic control and holonomic quantum computation in closed systems
- Principle of the adiabatic control
- Principle of the holonomic quantum computation
- Geometric framework

Quantum control and quantum information

$$\imath\hbar\dot{U}(t,0) = \underbrace{(H_0 + H_{ctrl}(x(t)))}_{H(x(t))} U(t,0) \qquad U(0,0) = 1 \quad U(t,0) \in \mathcal{U}(\mathcal{H})$$

$x \in M$ (control manifold).
Control problem : find a path $\mathscr{C} : [0,T] \ni t \mapsto x(t)$ such that $|\langle\psi_{target}|U(T,0)|\psi_0\rangle|^2 \simeq 1$, with a fixed initial state ψ_0 and a previously chosen target state ψ_{target}.
Targets associated with quantum information problems :

- $U(T,0)$ is a logical gate.
- $\psi_0 \to \psi_{target}$ corresponds to a transport of information.
- ψ_{target} is an entangled state.

Principle of the adiabatic transport

We assume that $\mathrm{Sp}(H(x))$ is pure point, without any degeneracy, and $\sharp\mathrm{Sp}(H(x)) = N < +\infty$.

$$H(x)\phi_n(x) = \lambda_n(x)\phi_n(x) \qquad \lambda_n, \phi_n \in \mathcal{C}^1(M)$$

If $T \gg \frac{\hbar}{\min_{n,p} \inf_{t\in[0,T]} |\lambda_n(x(t)) - \lambda_p(x(t))|}$ and if $\mathscr{C}$ is $\mathcal{C}^1$ then

$$U(t,0) = \sum_n e^{\imath \varphi_n(t)} |\phi_n(x(t))\rangle\langle\phi_n(x(0))| + \mathcal{O}(1/T)$$

with $\varphi_n(t) = -\hbar^{-1} \int_0^t \lambda_n(x(t'))dt' + \imath \int_{\mathscr{C}_t} A_n$ ($A_n = \langle\phi_n|d|\phi_n\rangle$, d being the exterior differential of M).

T. Kato, Phys. Soc. Jap. 5, 435 (1950)

Rapid passage by a conical crossing

We assume that :

- $\mathrm{Sp}(H(x))$ is non-degenerate except at $x_\times$ where $\lambda_n(x_\times) = \lambda_{n+1}(x_\times)$;
- $|\lambda_n(x(t)) - \lambda_{n+1}(x(t))| = \alpha|t_\times - t| + \mathcal{O}(|t_\times - t|^2)$ in the neighbourhood of $t_\times$ such that $x(t_\times) = x_\times$;
- $\mathscr{C}$ is only $\mathcal{C}^0$ at $x_\times$.

$$U(t_\times + \delta t, t_\times - \delta t) = \begin{pmatrix} \sin\xi & \cos\xi \\ -\cos\xi & \sin\xi \end{pmatrix}_{(\phi_n,\phi_{n+1})} + \mathcal{O}(1/\sqrt{T})$$

with $\delta t \sim \sqrt{\hbar/\alpha} \ll T$ and $\tan(2\xi) = \frac{\|\vec{\mathfrak{t}}(t_\times^-)\wedge\vec{\mathfrak{t}}(t_\times^+)\|}{\vec{\mathfrak{t}}(t_\times^-)\cdot\vec{\mathfrak{t}}(t_\times^+)}$ ($\vec{\mathfrak{t}}$ being the tangent vector at $\mathscr{C}$).

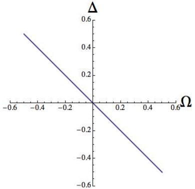
S. Teufel, Adiabtic perturbation theory in quantum dynamics (Springer, 2003)

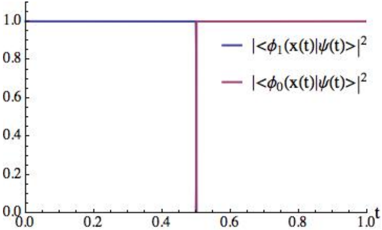
U. Boscain *etal.* IEEE Transactions on automatic control 57, 1970 (2012).

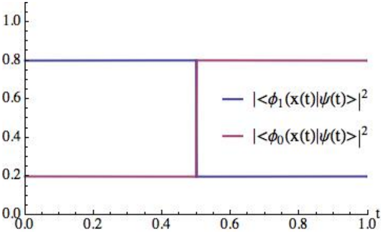
Example (pseudo NOT gate)

$$H(x) = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega \\ \Omega & \Delta \end{pmatrix}$$

$$x = (\Omega, \Delta) \in M = \mathbb{R}^2$$



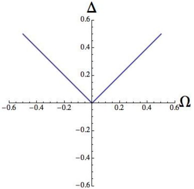


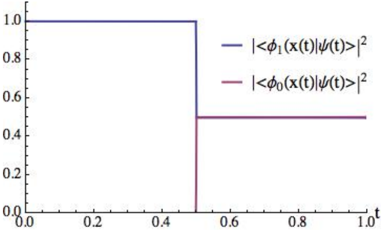


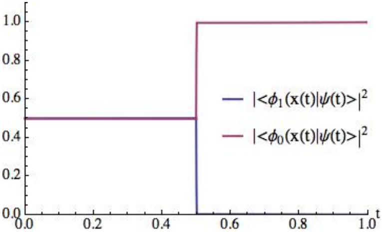
Example (pseudo Hadamard gate)

$$H(x) = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega \\ \Omega & \Delta \end{pmatrix}$$

$$x = (\Omega, \Delta) \in M = \mathbb{R}^2$$







The case of a globally degenerate eigenvalue

$$H(x)\phi_{n,a}(x) = \lambda_n(x)\phi_{n,a}(x) \qquad \forall x \in M, \quad \lambda_n, \phi_{n,a} \in \mathcal{C}^1(M)$$

$$U(t,0)P_a(x(0)) = e^{\imath\varphi_n^{dyn}} \sum_{ba} \left[\mathbb{P}e^{-\int_{\mathcal{C}_t} A_n}\right]_{ba} |\phi_{n,b}(x(t))\rangle\langle\phi_{n,a}(x(0))| + \mathcal{O}(1/T)$$

with $\varphi_n^{dyn} = -\hbar^{-1}\int_0^t \lambda(x(t'))dt'$ and

$$[A_n(x)]_{ab} = \langle\phi_{n,a}|d|\phi_{n,b}\rangle$$

Principle of the Holonomic Quantum Computation

The holonomy $H(\mathscr{C}) = \mathbb{P}e^{-\oint_{\mathscr{C}} A_n}$ (for a closed path $\mathscr{C}$) constitutes a "logic gate" for the qudit represented by the degenerate eigenspace. Under some small assumptions, $\mathrm{Hol}_{x_0} = \{H(\mathscr{C})\}_{\mathscr{C} \in \mathscr{L}_{x_0} M} = U(N)$ (Ambrose-Singer-Chow-Rashevski theorem).

P. Zanardi and M. Rasetti, Phys. Lett. A 264, 94 (1999)

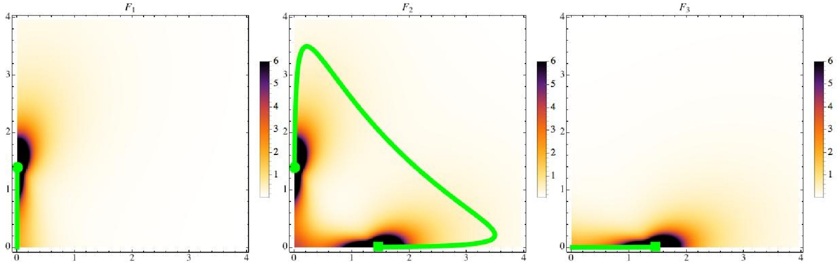
D. Lucarelli, J. Math. Phys. 46, 052103 (2005)

Geometric framework

- The adiabatic transport is described within a $U(N)$-principal bundle with base space M (the control manifold) and endowed with a connection described by the potential $A_n \in \Omega^1(M, \mathfrak{u}(N))$.
- $F_n = dA_n + A_n \wedge A_n$ constitutes the curvature of the fibre bundle.
- M can be embedded in a complex projective space.
- M can be endowed with some metrics $g_{\mu\nu}$ and/or some symplectic forms $\omega_{\mu\nu}$ (or with some Kähler forms $K_{\mu\nu}$).
- The controllability and the control robustness problems can be reformulated in a geometric language.

Example : F_n is a measure of the non-adiabatic effects

$$H(x) = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega_P & 0 \\ \Omega_P & 2\Delta_P & \Omega_S \\ 0 & \Omega_S & 2(\Delta_P - \Delta_S) \end{pmatrix}$$



2 The open quantum systems

- Description of the open quantum systems
- Effects induced by the environment

The density matrix

$$\Psi \in \mathcal{H}_\mathcal{S} \otimes \mathcal{H}_\mathcal{E} \Rightarrow \rho = \mathrm{tr}_\mathcal{E} |\Psi\rangle\rangle\langle\langle\Psi|$$

$$\rho^\dagger = \rho, \quad \mathrm{tr}\rho = 1, \quad \rho > 0$$

ρ is a pure state if $\rho^2 = \rho$, else it is called a mixed state.

$$\imath\hbar\dot{\rho} = [H_0 + H_{ctrl}(x(t)), \rho(t)] + \mathrm{tr}_\mathcal{E}\, [H_{int}, |\Psi\rangle\rangle\langle\langle\Psi|]$$

with $H_{int} \in \mathcal{L}(\mathcal{H}_\mathcal{S} \otimes \mathcal{H}_\mathcal{E})$.
Control problem : find a path $\mathscr{C} : [0,T] \ni t \mapsto x(t)$ such that $\mathrm{tr}\left(\rho^\dagger_{target}\rho(T)\right) \simeq 1$.

The Lindblad equation

Under the assumptions :

- $\mathcal{E}$ is a very large stationary environment (as a thermal bath for example).
- The coupling between $\mathcal{S}$ and $\mathcal{E}$ is sufficiently small (Born approximation).
- (The correlation time of the bath is smaller than the caracteristic time of the interaction system-bath (Markovian regime))

without control we have :

$$\imath\hbar\dot{\rho} = [\underbrace{H_0 + H_{LS}}_{H_1}, \rho] - \frac{\imath}{2}\sum_k \gamma_k\{\Gamma_k^\dagger\Gamma_k, \rho\} + \imath\sum_k \gamma_k\Gamma_k\rho\Gamma_k^\dagger$$

H.-P. Breuer and F. Petruccione, Open quantum systems (Oxford University Press, 2002).

The Hilbert-Schmidt (Liouville) representation

For a two level system, in the secular approximation ($\tau_{\mathcal{S}} \ll \tau_R$) : $\{\Gamma_k\}_k = \{\sigma_+, \sigma_-, \sigma_z\}$.

$$\rho = \sum_{i,j=0}^{1} \rho_{ij} |i\rangle\langle j| \xrightarrow{HS} |\rho\rangle\rangle = \rho_{ij} |i\rangle \otimes |j\rangle$$

$$\imath\hbar|\dot{\rho}\rangle\rangle = L|\rho\rangle\rangle \qquad (L^\dagger \neq L) \quad \text{with } L = L_d \oplus L_r$$

$$L_d = \begin{pmatrix} \Delta\lambda - \imath\frac{\Gamma}{2} & 0 \\ 0 & -\Delta\lambda - \imath\frac{\Gamma}{2} \end{pmatrix}_{(|10\rangle\rangle, |01\rangle\rangle)}$$

$$L_r = \begin{pmatrix} -\imath\gamma_- & \imath\gamma_+ \\ \imath\gamma_- & -\imath\gamma_+ \end{pmatrix}_{(|00\rangle\rangle, |11\rangle\rangle)}$$

$(\Delta\lambda = \lambda_1 - \lambda_0,\ \Gamma = \gamma_+ + \gamma_- + \gamma_z,\ H_1|i\rangle = \lambda_i|i\rangle)$

Purification of the dynamics

Let $\Psi_\rho \in \mathcal{H}_\mathcal{S} \otimes \mathcal{H}_\mathcal{A}$ ($\mathcal{H}_\mathcal{A}$ is an ancilla (with $\dim \mathcal{H}_\mathcal{A} = \dim \mathcal{H}_\mathcal{S}$) playing the role of an effective small environment), be such that

$$\rho = \mathrm{tr}_\mathcal{A} |\Psi_\rho\rangle\!\rangle \langle\!\langle \Psi_\rho|$$

$$\imath\hbar\dot{\Psi}_\rho = \left(H_1 - \frac{\imath}{2} \sum_k \gamma_k \Gamma_k^\dagger \Gamma_k \right) \otimes 1_\mathcal{A} \Psi_\rho + \frac{\imath}{2} \sum_k \gamma_k \Gamma_k \otimes \Gamma_k^\ddagger(\Psi_\rho) \Psi_\rho$$

with $\Gamma_k^\ddagger(\Psi_\rho) = (W_\rho^\dagger \Gamma_k^\dagger (W_\rho^\dagger)^{-1})^\mathsf{T}$ ($W_\rho = \sum_{i,j} \Psi_{\rho,ij} |i\rangle\langle j|$).

D. Viennot, arXiv :1508.02279 (2015).

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Stochastic Schrödinger equation representation

In the Markovian regime, we have

$$\imath\hbar d\psi = \left(H_1 - \frac{\imath}{2}\sum_k \gamma_k(\Gamma_k^\dagger\Gamma_k - \langle\Gamma_k^\dagger\Gamma_k\rangle_\psi)\right)\psi dt + \imath\sum_k\left(\frac{\Gamma_k}{\|\Gamma_k\psi\|} - 1\right)\psi dN_{k,t}$$

where $dN_{k,t}$ is a Poisson process with $\mathbb{E}[dN_{k,t}] = \gamma_k\|\Gamma_k\psi\|^2 dt$. $N_{k,t}$ counts the number of jumps of type Γ_k.

$$\rho = \mathbb{E}\left[|\psi\rangle\langle\psi|\right]$$

H.-P. Breuer and F. Petruccione, Open quantum systems (Oxford University Press, 2002).

Dynamical effects induced by the environment

- **Decoherence** : under the action of L_d, $\rho_{10}(t) \to 0$.
- **Relaxation** : under the action of L_r, $\rho_{ii}(t) \to \frac{\gamma_\pm}{\gamma_+ + \gamma_-}$.
- **Dissipation** : under only the action of $-\frac{\imath}{2} \sum_k \gamma_k \Gamma_k^\dagger \Gamma_k \otimes 1_{\mathcal{A}}$, $\|\Psi_\rho\| \to 0$.
- **Entanglement** : under the action of $\frac{\imath}{2} \sum_k \gamma_k \Gamma_k \otimes \Gamma_k^\ddagger(\Psi_\rho)$ the ancilla and the system become entangled.
- **Quantum jumps** : under the action of $\frac{\imath}{2} \sum_k \gamma_k \Gamma_k \otimes \Gamma_k^\ddagger(\Psi_\rho)$ or $\imath \sum_k \left(\frac{\Gamma_k}{\|\Gamma_k \psi\|} - 1 \right) dN_t^k$ the Hilbert-Schmidt norm is restored.

Control of an open quantum system

$$\imath\hbar\dot{\Psi} = \left((H_0 + H_{ctrl}(x(t))) \otimes 1_{\mathcal{E}} + 1_{\mathcal{S}} \otimes H_{\mathcal{E}} + H_{int}\right) \Psi$$

$$\begin{aligned}\longrightarrow \imath\hbar\dot{\rho} \quad = \quad & [H_0 + H_{ctrl}(x(t)) + H_{LS}(u(t)), \rho] \\ & -\frac{\imath}{2}\sum_k \gamma_k(u(t))\{\Gamma_k(u(t))^\dagger \Gamma_k(u(t)), \rho\} \\ & +\imath \sum_k \gamma_k(u(t)) \Gamma_k(u(t)) \rho \Gamma_k(u(t))^\dagger\end{aligned}$$

with $\imath\hbar\dot{u}(t) = (H_0 + H_{ctrl}(x(t)))u(t)$, $u(0) = 1_{\mathcal{S}}$, $u(t) \in \mathcal{U}(\mathcal{H}_{\mathcal{S}})$.

Hampering of the control by the environment

- **Decoherence** : the environment kills the coherences.
- **Relaxation** : the environment forces the populations to go to the steady state populations.
- **Dissipation** : the environment kills all states (except the steady state).
- **Entanglement** : the system becomes entangled with the environment.
- **Back-reaction** : the dynamical effects induced by the environment react against the control.
- **Distorsion** : the environment adds noises on the control signal ($x(t) \to x(t) + \delta x_{\mathcal{E}}(t)$).

Simplified models

- Only decoherence : "phase-damping" models ($\Gamma_k^2 = \Gamma_k$, etc).
- Only dissipation : $\imath\hbar\dot{\psi} = (H - \imath D)\psi$ ($H^\dagger = H$, $D^\dagger = D$) (non-hermitian quantum dynamics).
- Only relaxation : $\imath\hbar\dot{P} = [H, P] - \imath\{D, P\} + 2\imath \mathrm{tr}(DP)P$ ($P = |\psi\rangle\langle\psi|/\|\psi\|^2$ with $\imath\hbar\dot{\psi} = (H - \imath D)\psi$).
- Only entanglement : small environment with $\imath\hbar\dot{\Psi} = (H_{\mathcal{S}}(x(t)) \otimes 1_{\mathcal{E}} + 1_{\mathcal{S}} \otimes H_{\mathcal{E}}(x(t)) + H_{int}(x(t)))\Psi$ (bipartite quantum system).
- Only distorsion : $\imath\hbar\dot{\psi} = H(x(t) + \delta x(t))\psi$ with $\delta x(t)$ noises (random, stochastic or chaotic processes), $\rho(t) = \mathbb{E}\left[|\psi(t)\rangle\langle\psi(t)|\right]$ ($\mathbb{E}$ being the expectation value with respect to the noises).

3 Adiabatic approaches for the open quantum systems

- Adiabatic regimes
- Disturbed control of quantum systems
- Non-hermitian adiabatic dynamics
- Adiabatic control hampered by entanglement in bipartite systems (small environments)
- Adiabatic control of a Lindbladian dynamics

Not only one adiabatic regime

Regime	$\mathcal{S}$ alone is adiabatic w.r.t. the control $\tau_{\mathcal{S}} \ll T$	$\mathcal{E}$ is adiabatic w.r.t. the control (or is stationary) $\tau_{\mathcal{E}} \ll T$	interaction is adiabatic w.r.t. the control $\tau_{int} \ll T$	$\mathcal{S}$ is adiabatic w.r.t. the interaction $\tau_{\mathcal{S}} \ll \tau_{int}$
barely adiabatic	✔	✘	✘	✘
very weakly adiabatic	✔	✔	✘	✘
weakly adiabatic	✔	✔	✘	✔
strongly adiabatic	✔	✔	✔	✘
very strongly adiabatic	✔	✔	✔	✔

A model : an atom in a laser field with phase noise

$$H(\theta+\delta\theta) = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega e^{\imath(\theta+\delta\theta)} \\ \Omega e^{-\imath(\theta+\delta\theta)} & 2\Delta \end{pmatrix} \qquad \delta\dot{\theta} = k(\theta(t))\eta(t)$$

η is a Gaussian white noise et $k(\theta+2\pi) = -k(\theta)$. The weak adiabatic transport (with $\psi(0) = \phi_0(\theta(0))$) is obtained by

$$\psi(t) \simeq e^{\imath\varphi_0(t)} W(\delta\theta(t))\phi_0(\theta(t))$$

with $W(\delta\theta(t)) = e^{-\frac{\imath\hbar}{2}\delta\theta(t)}|0\rangle\langle 0| + e^{\frac{\imath\hbar}{2}\delta\theta(t)}|1\rangle\langle 1|$ and
$\varphi_0(t) = -\hbar^{-1}\int_0^t \lambda_0(\theta(t'))dt' + \imath\int_0^t \langle\phi_0|\partial_{t'}|\phi_0\rangle dt' + \imath\int_0^t \langle\phi_0|W^{-1}\dot{W}|\phi_0\rangle dt'$.

$$\begin{aligned} \rho_{01}(t) &= \langle 0|\mathbb{E}\left[|\psi(t)\rangle\langle\psi(t)|\right]|1\rangle \\ &= e^{-\frac{\hbar^2}{4}K(t,0)}\langle 0|\phi_0(\theta(t))\rangle\langle\phi_0(\theta(t))|1\rangle \end{aligned}$$

$K(t,0) = D\int_0^t k(\theta(t'))^2 dt'$, $D\delta(t_2 - t_1) = \mathbb{E}[\eta(t_1)\eta(t_2)]$.

D. Viennot, J. Math. Phys. 53, **082106 (2012).**

A model : spin chains kicked by trains of ultrashort pulses

$$\begin{aligned} H(t) &= \frac{\hbar\omega_1}{2}\sum_{i=1}^{N} S_{z,i} \\ &\quad - \sum_{i=1}^{N}(J_x S_{x,i}S_{x,i+1} + J_y S_{y,i}S_{y,i+1} + J_z S_{z,i}S_{z,i+1}) \\ &\quad + \hbar \sum_{i=1}^{N} |w_i\rangle\langle w_i| \sum_{k\in\mathbb{N}} \lambda_i^{(k)}\delta(t-kT+\tau_i^{(k)}) \end{aligned}$$

where $(\lambda_i^{(k)},\tau_i^{(k)})_{k\in\mathbb{N}} = (\lambda_i^{ctrl,(k)},\tau_i^{ctrl,(k)})_{k\in\mathbb{N}} + (\delta\lambda_i^{(k)},\delta\tau_i^{(k)})_{k\in\mathbb{N}}$ is a (discrete time) evolution with $(\delta\lambda_i^{(k)},\delta\tau_i^{(k)})_{k\in\mathbb{N}}$ a chaotic noise.

$$\rho^{(k)} = \rho(kT) = \frac{1}{N}\sum_{i=1}^{N} \mathrm{tr}_{\neq i}\left(U(kT,0)|\psi_0\rangle\!\rangle\langle\!\langle\psi_0|U(kT,0)^\dagger\right)$$

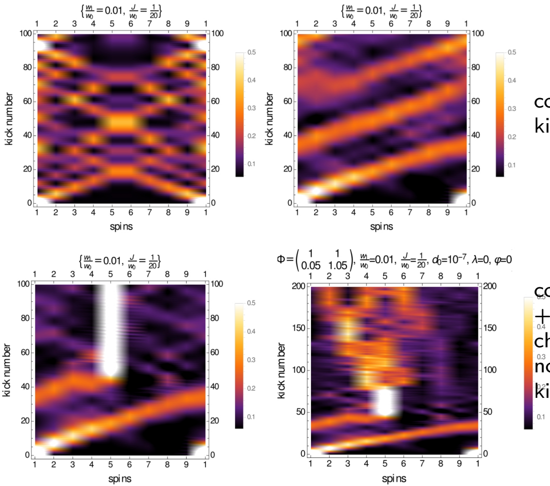
Control of the information transmission

without kicks

control kicks

control + chaotic noise kicks

control + chaotic noise kicks



L. Aubourg and D. Viennot, arXiv :1402.2411 (2016).

Non-hermitian (strong) adiabatic transport

Non-real eigenvalues :

$$H(x)\phi_n(x) = \lambda_n(x)\phi_n(x)$$

$$H(x)^\dagger \phi_n^\star(x) = \overline{\lambda_n(x)}\phi_n^\star(x)$$

$$\langle \phi_n^\star(x)|\phi_p(x)\rangle = \delta_{np}$$

$$U(t,0) = \sum_n e^{\imath\varphi_n(t)}|\phi_n(x(t))\rangle\langle\phi_n^\star(x(0))| + \mathcal{O}(e^{\Gamma_n t}/T)$$

with $\varphi_n(t) = -\hbar^{-1}\int_0^t \lambda_n(x(t'))dt' - \int_{\mathscr{C}_t} A_n \in \mathbb{C}$ ($A_n = \langle\phi_n^\star|d|\phi_n\rangle$), and $\Gamma_n = \Im\mathrm{m}(\lambda_0 - \lambda_n)$ ($\Im\mathrm{m}\lambda_{n+1} < \Im\mathrm{m}\lambda_n$).

A. Joye, Commun. Math. Phys. 275 139 (2015).

Weak adiabatic regime

$$U(t,0) = \sum_n e^{\imath \varphi_n(t)} \Omega_n(t) |\phi_n(x(t))\rangle\langle\phi_n^\star(x(0))|$$

with $\varphi_n(t) = -\hbar^{-1}\int_0^t \lambda_n(x(t'))dt' + \imath \int_{\mathscr{C}_t} A_n + \imath \int_0^t \langle\phi_n|\dot{\Omega}_n|\phi_n\rangle dt \in \mathbb{C}$.
$\Omega_n(t)$ is a time-dependent wave operator :

$$(H(x(t)) - \imath\hbar\partial_t)\Omega_n(t) = \Omega_n(t)(H(x(t)) - \imath\hbar\partial_t)\Omega_n(t)$$

$$\Omega_n(t)P_n(t) = \Omega_n(t) \qquad P_n(t)\Omega_n(t) = P_n(t)$$

D. Viennot, J. Phys. A 47, 065302 (2014).

Exceptional points

$x_\times$ is an exceptional point (EP) if

- two eigenvalues crosses in the complex plane : $\lambda_n(x_\times) = \lambda_{n+1}(x_\times)$ (algebraic multiplicity equal to 2) ;
- we have a coalescence of the associated eigenvectors (geometric multiplicity equal to 1) :

$$\lim_{x \to x_\times} \phi_n(x) \propto \lim_{x \to x_\times} \phi_{n+1}(x)$$

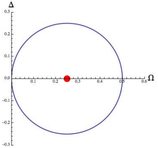
- $H(x)$ presents a Jordan bloc at $x_\times$:

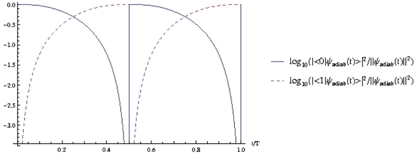
$$H(x_\times) = \begin{pmatrix} \lambda_n(x_\times) & 1 \\ 0 & \lambda_n(x_\times) \end{pmatrix}_{(\phi_n(x_\times), \chi_n)}$$

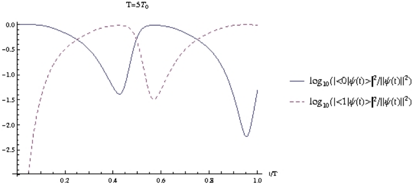
for a good chosen generalized eigenvector χ_n.

Example : surrounding an EP

$$H(x) = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega \\ \Omega & 2\Delta - \imath \frac{\Gamma}{2} \end{pmatrix}$$



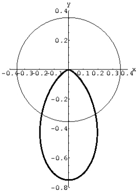


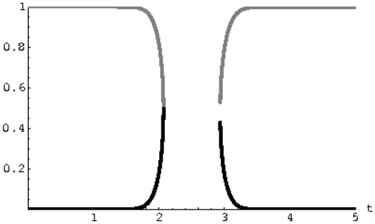


$(\|\psi(T)\| \simeq 10^{-13})$

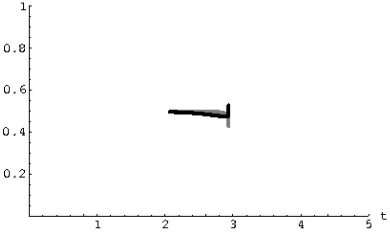
Example : passage through an EP

$$H(x) = \frac{\hbar}{2} \begin{pmatrix} 0 & x - \imath y \\ x + \imath y & 2\Delta - \imath \frac{\Gamma}{2} \end{pmatrix}$$





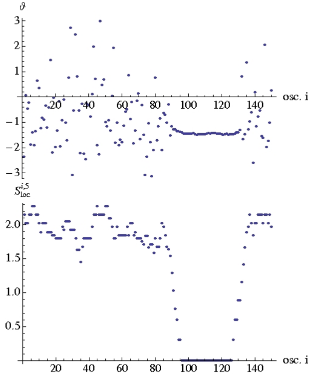
interior state populations



exterior state populations

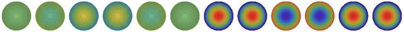
Other models : spin chain with quantum chimera states

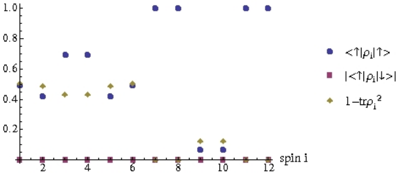
classical chimera state (oscillator chain)

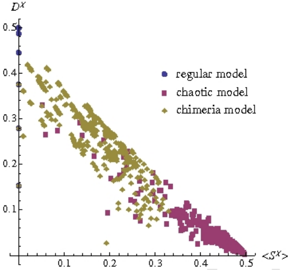


D. Viennot and L. Aubourg, Phys. Lett. A 380, 678 (2016).

quantum chimera state (spin chain)







$$\langle S \rangle = \langle 1 - \mathrm{tr}\rho^2 \rangle$$
$$D = 1 - \mathrm{tr}\langle \rho \rangle^2 - \langle S \rangle$$

Weak adiabatic transport

Non-commutative eigenvalues :

$$H_{\mathcal{S}+\mathcal{E}}(x)\Phi_E(x) = E(x)\otimes 1_{\mathcal{E}}\Phi_E(x) \qquad E \in \mathcal{L}(\mathcal{H}_{\mathcal{S}}), \quad \Phi_E(x) \in \mathcal{H}_{\mathcal{S}}\otimes\mathcal{H}_{\mathcal{E}}$$

$$[H_{\mathcal{S}+\mathcal{E}}(x), E(x)\otimes 1_{\mathcal{E}}]\Phi_E(x) = 0$$

Let $\rho_E(x) = \mathrm{tr}_{\mathcal{E}}|\Phi_E(x)\rangle\rangle\langle\langle\Phi_E(x)|$.
If $\rho(0) = \rho_E(x(0))$, then

$$\rho(t) = \mathfrak{Ad}\left[\underset{\leftarrow}{\mathbb{T}e}^{-\imath\hbar^{-1}\int_0^t E(x(t'))dt'}\underset{\rightarrow}{\mathbb{P}e}^{-\int_{\mathcal{C}_t} A}\right]\rho_E(x(t)) + \mathcal{O}(\max(1/T, 1/\tau_{int}))$$

with $\mathfrak{Ad}[g]\rho = g\rho g^\dagger$, $d\rho_E = A\rho_E + \rho_E A^\dagger$ $(A \in \mathcal{L}(\mathcal{H}_{\mathcal{S}}))$.

(The strong adiabatic transport is obtained with $E(x) = \lambda(x)1_{\mathcal{H}_{\mathcal{S}}}$).

D. Viennot and J. Lages, J. Phys. A 44, 365301 (2011).

D. Viennot and L. Aubourg, J. Phys. A 48, 025301 (2015).

Geometric framework

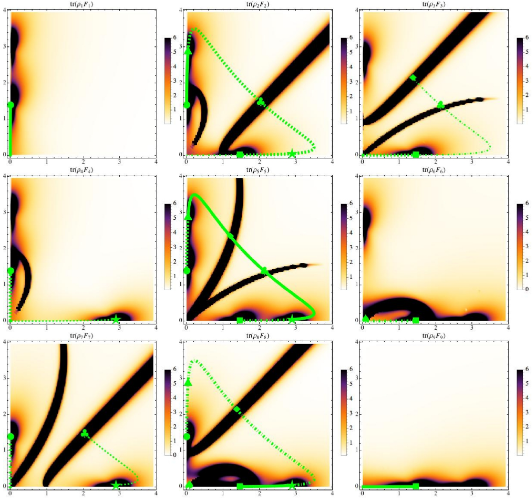
The geometric structure is not a principal bundle but a two-sided categorical bundle with a 2-connection :

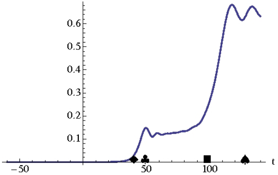
Connective structure		*interpretation*
left potential	A	generator of the operator-valued geometric phases
right object potential	A^R_o	generator of the Uhlmann geometric phases
right arrow potential	$A^R_\to$	unitary and SLOCC operations on $\mathcal{S}$
left curving	B^L	measure of the purity decreasing effects
right curving	B^R	measure of the shift in the standard purification
left fake curvature	F^L	measure of the non-adiabatic and entangl. effects
right fake curvature	F^R	quantity related to the quantum Fisher information
two-sided fake curvature	F^{RL}	measure of the non-invariance

D. Viennot, arXiv :1508.02279 (2015).

Example : Left fake curvature (non-adiab. trans. entangl.)

$\mathcal{S}$ and $\mathcal{E}$ are two coupled 3-level atoms in laser fields.





$$S(t) = -\mathrm{tr}(\rho(t)\ln\rho(t))$$
$$\rho = \mathrm{tr}_{\mathcal{E}}|\Psi\rangle\rangle\langle\langle\Psi|$$

D. Viennot, J. Phys. A 47, 295301 (2014).

Master equation in the weak adiabatic assumption

$$H(x)\phi_n(x) = \lambda_n(x)\phi_n(x)$$

$$U_{\mathcal{S}}(t,t_0) \simeq \sum_n e^{\imath\varphi_n(t)}|\phi_n(x(t))\rangle\langle\phi_n(x(t_0))| \quad (t_\times \notin [t,t_0])$$

$$\begin{aligned}\longrightarrow \imath\hbar\dot{\rho} &= [H(x(t)),\rho] \\ &\quad -\frac{\imath}{2}\sum_k \gamma_k(R(x(t)),\xi)\{R(x(t))\Gamma_k^\dagger\Gamma_k R(x(t))^\dagger,\rho\} \\ &\quad +\imath\sum_k \gamma_k(R(x(t)),\xi)R(x(t))\Gamma_k R(x(t))^\dagger\rho R(x(t))\Gamma_k^\dagger R(x(t))^\dagger\end{aligned}$$

with $R(x) = \sum_n |\phi_n(x)\rangle\langle\phi_n(x_0)|$ and ξ the mixing angle at the crossing.
$\gamma_k(R(x(t)),\xi)$ in Markovian and non-Markovian cases? (work in progress)

Adiabatic transport with Lindblad equation

- The strong adiabatic transport can be obtained as a non-hermitian adiabatic transport in the Hilbert-Schmidt representation of the Lindblad dynamics (work in progress for the Markovian secular case with a 3-level atom in a thermal bath).
- The weak adiabatic transport could be obtained as the non-commutative adiabatic transport in the purified representation of the Lindblad dynamics **but** the nonlinearity of the purified equation is a strong obstruction.