

**Phase of biparticle localized states for the Cooper problem in two-dimensional disordered systems**

J. Lages and D. L. Shepelyansky\*

*Laboratoire de Physique Quantique, UMR 5626 du CNRS, Université Paul Sabatier, F-31062 Toulouse Cedex 4, France*

(Received 11 April 2001; published 31 July 2001)

The Cooper problem is studied numerically for the Anderson model with disorder in two dimensions. It is shown that the attractive Hubbard interaction creates a phase of biparticle localized states in the regime where noninteracting states are delocalized. This phase cannot be obtained in the mean-field approximation and the pair coupling energy is strongly enhanced in this regime. The effects of magnetic field are studied and it is shown that under certain conditions they lead to delocalization.

DOI: 10.1103/PhysRevB.64.094502

PACS number(s): 74.20.-z, 74.25.-q, 74.40.+k

**I. INTRODUCTION**

Recently a great deal of attention has been paid to investigation of the superconductor-insulator transition (SIT) in systems with disorder. Various approaches are used to study this problem including analytical theoretical methods,<sup>1,2</sup> intensive numerical simulations of many-body quantum systems with quantum Monte Carlo methods,<sup>3,4</sup> as well as mean field numerical simulations.<sup>5-7</sup> These theoretical studies are stimulated by challenging experiments on the SIT in disordered films<sup>8,9</sup> and high- $T_c$  superconductors.<sup>10,11</sup> The results obtained in Ref. 10 show an interesting correlation between the optimal doping and the Anderson transition in the normal phase obtained by application of a strong pulsed magnetic field. Even if the experiments<sup>10,11</sup> are done with three-dimensional crystals the coupling between two-dimensional planes is relatively weak and the two-dimensional effects should play an important role. Due to that, it is relevant to study the SIT in two-dimensional disordered systems. In the case of weak disorder the Anderson theorem<sup>12,13</sup> guarantees that the superconductivity is not affected by disorder. However it is not obvious if the theorem is still valid in the presence of relatively strong disorder. It is quite possible that in this regime the interplay of disorder and interaction can lead to the appearance of new physical effects. The theoretical investigation of this regime is however rather difficult. The existing analytical methods are not well adapted to the regime of strong interaction and disorder. At the same time the numerical studies also meet with serious difficulties. Indeed the direct diagonalization methods are restricted to relatively small system size since the Hilbert space grows exponentially with the number of particles.<sup>14,15</sup> The quantum Monte Carlo methods are not so sensitive to a huge size of the Hilbert space but still they are restricted to systems of quite moderate size (for example lattices of  $8 \times 8$  sites in Ref. 4).

In view of the above numerical difficulties it is natural to develop the approach introduced by Cooper<sup>16</sup> and to study the problem of two particles with attractive interaction near the frozen Fermi sea in the presence of disorder. Even if the original Cooper problem of two particles without disorder does not reproduce exactly the BCS theory of the many-body problem it nevertheless captures the essential physical properties of the system and gives the appearance of coupled

states with a qualitatively correct coupling energy and correlation length. Without disorder the Cooper problem can be solved exactly. However in the presence of disorder the situation becomes more complicated even for only two interacting particles (TIP). Indeed, for relatively strong disorder even the matrix elements of interaction between noninteracting eigenstates cannot be obtained analytically and due to that the problem should be studied numerically. The first numerical studies of the Cooper problem in the presence of disorder were done in Ref. 17 for two particles with attractive Hubbard interaction in the three-dimensional Anderson model. These studies showed that the interaction can lead to localization of pairs in the noninteracting metallic phase. This result is qualitatively different from the mean-field solution of the Cooper problem in the presence of disorder (Cooper ansatz), which gives delocalized pairs for the same parameters. This shows that the nondiagonal interaction-induced matrix elements play an important role and lead to new physical effects, which are not captured by the mean-field approximation.

In this paper we study the Cooper problem on a two-dimensional lattice with disorder described by the Anderson model. Our numerical studies show that near the Fermi level the attractive Hubbard interaction between two particles creates localized pairs in the regime where noninteracting eigenstates are well delocalized (extended). The coupling energy of these pairs is much larger than the coupling energy given by the mean-field solution (Cooper ansatz). Therefore energetically it is more favorable to have an insulator with localized pairs instead of usual weakly coupled delocalized Cooper pairs. This result indicates the appearance of a new phase of biparticle localized states (BLS phase), which appears in the regime when noninteracting states are extended (metallic). It is in a qualitative agreement with the quantum Monte Carlo studies obtained recently in Toulouse.<sup>18</sup> This BLS phase is qualitatively different from the BCS solution, which corresponds to weakly coupled delocalized pairs.

The paper is organized as follows. The properties of the BLS phase without magnetic field are discussed in Sec. II. The effects of perpendicular magnetic field on the ground-state properties in the presence of interaction and disorder are analyzed in Sec. III. The discussion of the results is presented in the last section.

## II. GROUND-STATE PROPERTIES WITHOUT MAGNETIC FIELD

To study the Cooper problem of two interacting particles in the presence of disorder we use the two-dimensional Anderson model. In this model the one-particle eigenstates are determined by the Hamiltonian

$$H_1 = \sum_{\mathbf{n}} E_{\mathbf{n}} |\mathbf{n}\rangle \langle \mathbf{n}| + V \sum_{\langle \mathbf{n}, \mathbf{m} \rangle} |\mathbf{n}\rangle \langle \mathbf{m}|, \quad (1)$$

where  $\mathbf{n}$  and  $\mathbf{m}$  are index vectors on the two-dimensional square lattice with periodic boundary conditions,  $V$  is the nearest-neighbor hopping term, and the random on-site energies  $E_{\mathbf{n}}$  are homogeneously distributed in the energy interval  $[-W/2, W/2]$ , where  $W$  is the disorder strength. This one-particle model has been extensively studied by different authors, see for example Ref. 19. For the two-particle problem on this two-dimensional lattice we consider on-site attractive Hubbard interaction between particles of strength  $U < 0$ . We consider the particles in the singlet state with zero total spin so that the spatial wave function is symmetric with respect to particle permutation (interaction is absent in the triplet state).

To investigate the effects of interaction between particles near the Fermi level we generalize the Cooper approach for the case with disorder. To do that we rewrite the TIP Hamiltonian in the basis of one-particle eigenstates of the Hamiltonian (1). In this basis the Schrödinger equation for TIP reads

$$(E_{m_1} + E_{m_2}) \chi_{m_1, m_2} + U \sum_{m'_1, m'_2} Q_{m_1, m_2, m'_1, m'_2} \chi_{m'_1, m'_2} = E \chi_{m_1, m_2}. \quad (2)$$

Here  $E_m$  are the one-particle eigenenergies corresponding to the one-particle eigenstates  $|\phi_m\rangle$  and  $\chi_{m_1, m_2}$  are the components of the TIP eigenstate in the noninteracting eigenbasis  $|\phi_{m_1}, \phi_{m_2}\rangle$ . The matrix elements  $U Q_{m_1, m_2, m'_1, m'_2}$  give the interaction-induced transitions between noninteracting eigenstates  $|\phi_{m_1}, \phi_{m_2}\rangle$  and  $|\phi_{m'_1}, \phi_{m'_2}\rangle$ . These matrix elements are obtained by rewriting the Hubbard interaction in the noninteracting eigenbasis of model (1). In the analogy with the original Cooper problem<sup>16</sup> the summation in Eq. (2) is done over the states above the Fermi level with eigenenergies  $E_{m'_1, 2} > E_F$  with  $m'_{1, 2} > 0$ . The Fermi energy  $E_F \approx 0$  is determined by a fixed filling factor  $\nu = \frac{1}{2}$ . To keep the similarity with the Cooper problem we restrict the summation on  $m'_{1, 2}$  by the condition  $1 < m'_1 + m'_2 \leq M$ . In this way the cutoff with  $M$  unperturbed orbitals introduces an effective phonon frequency  $\omega_D \propto M/L^2 = 1/\alpha$  where  $L$  is the linear system size. When varying  $L$  we keep  $\alpha$  fixed so that the phonon frequency is independent of system size. All the data in this work are obtained with  $\alpha = 15$  but we also checked that the results are not sensitive to the change of  $\alpha$ . We note that a similar TIP model was considered for the problem of two repulsive quasiparticles near the Fermi level in Refs. 20 and 21. However, there the studies mainly addressed the proper-

ties of excited states while here we will investigate only the properties of the ground state in the case of an attractive interaction. We also note that the attractive case in one dimension was discussed in Refs. 22 and 23.

To determine the characteristics of the ground state of the generalized Cooper problem (2) we solve numerically the Schrödinger equation. After that we rewrite the obtained ground state in the original lattice basis with the help of the relation between the lattice basis and one-particle eigenstates  $|\mathbf{n}\rangle = \sum_m R_{\mathbf{n}, m} |\phi_m\rangle$ . As a result of this procedure we obtain the two-particle probability distribution  $F(\mathbf{n}_1, \mathbf{n}_2)$  from which we extract the one-particle probability  $f(\mathbf{n}) = \sum_{\mathbf{n}_2} F(\mathbf{n}_1, \mathbf{n}_2)$  and the probability of interparticle distance  $f_d(\mathbf{r}) = \sum_{\mathbf{n}_2} F(\mathbf{r} + \mathbf{n}_2, \mathbf{n}_2)$  with  $\mathbf{r} = \mathbf{n}_1 - \mathbf{n}_2$ .

Typical examples of such probability distributions are presented in Fig. 1. Without interaction, at given disorder strength  $W = 2V$  and  $W = 5V$  both particles are delocalized on the lattice of given size  $L = 40$ . In the presence of interaction  $U = -2V$  the ground state remains delocalized for  $W = 2V$  and the probability distribution is rather similar to the case of  $U = 0$  [compare Figs. 1(a) and 1(c)]. On the contrary for  $W = 5V$ , interaction completely changes the ground-state properties leading to a clear localization of both particles near each other [compare Figs. 1(b) and 1(e)]. The wave function is localized in a rather compact way and the finite size of the lattice definitely does not affect this localization. Figure 1(f) shows that in this localized state the particles remain correlated close to each other. This biparticle localized ground state is obtained by exact diagonalization of Eq. (2) where all nondiagonal interaction-induced matrix elements are taken into account. It is interesting to compare this solution with the mean-field approximation (Cooper ansatz) in which only diagonal terms are taken into account. Within the Cooper ansatz the particles occupy the same noninteracting orbitals and only matrix elements  $Q_{m_1, m_2, m'_1, m'_2}$  with  $m_1 = m_2$  and  $m'_1 = m'_2$  are kept in Eq. (2). The ground state obtained from the Cooper ansatz is shown in Figs. 1(g) and 1(h) and is clearly delocalized contrarily to the strongly localized ground state obtained from exact diagonalization of Eq. (2) and shown in Figs. 1(e) and 1(f). In fact the ground state from the Cooper ansatz is closer to the delocalized noninteracting eigenstate in Fig. 1(b) than to the real eigenstate in Fig. 1(e) in the presence of interaction. The results of Fig. 1 definitely show that the attractive interaction leads to localization of pairs in the regime when noninteracting states are delocalized. This localization is not captured by the Cooper ansatz, which neglects nondiagonal matrix elements and due to that misses the essential physical effect.

In order to study the ground-state properties of our model in a more quantitative way it is convenient to compute the inverse participation ratio (IPR)  $\xi$  defined as  $\xi^{-1} = \langle \sum_{\mathbf{n}} f^2(\mathbf{n}) \rangle$  where the brackets mark the averaging over  $N_D$  disorder realizations (typically  $N_D = 100$ ). Physically,  $\xi$  gives the number of lattice sites occupied by one particle in the TIP ground state. The dependence of the IPR  $\xi$  on the disorder strength  $W$  is shown in Fig. 2 for different strengths of interaction  $U$  and different system sizes  $L$ . In the absence of interaction, for finite system sizes used in our numerical

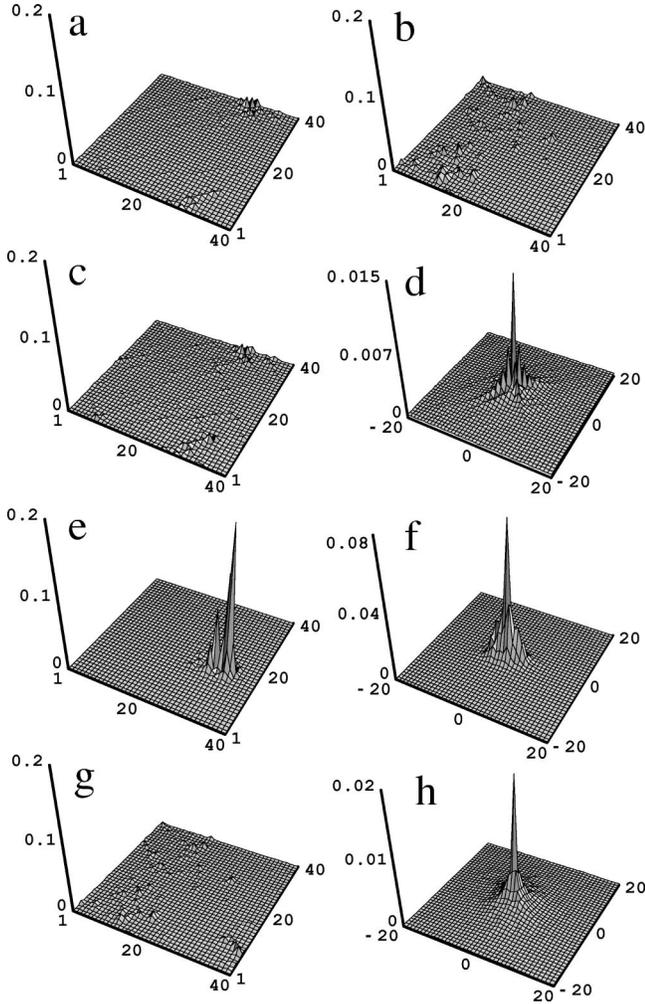


FIG. 1. Ground-state probability distributions of two interacting particles for the Cooper problem with disorder on a lattice of linear size  $L=40$ . The cases (a) and (b) show the one-particle probability distribution  $f(\mathbf{n})$  in the absence of interaction ( $U=0V$ ) for the disorder strength  $W=2V$  (a) and  $W=5V$  (b). All other cases are obtained for the Hubbard interaction  $U=-2V$ . The cases (c) and (d) show the one-particle probability  $f(\mathbf{n})$  (c) and the interparticle distance probability  $f_d(\mathbf{r})$  (d) for  $W=2V$ . The same probabilities are shown in cases (e) and (f) for  $W=5V$  [ $f(\mathbf{n})$  for (e) and  $f_d(\mathbf{r})$  for (f)]. The cases (g) and (h) present the probabilities for the same  $W=5V$  as in cases (e) and (f) however here the ground state is obtained in the mean-field approximation of the Cooper ansatz. All data are given for the same realization of disorder.

simulations ( $L \leq 40$ ), the ground state is delocalized for disorder  $W \leq 5V$  and it becomes localized for  $W > 5V$ . On the contrary in the presence of interaction the TIP ground state becomes localized for  $W > W_c \approx 2V$  at  $U = -4V$  and for  $W > W_c \approx 3V$  at  $U = -2V$ . Indeed for  $W < W_c$  the IPR  $\xi$  starts to grow significantly with the increase of the system size  $L$  that corresponds to pair delocalization. The decrease of the  $W_c$  value induced by the attractive interaction shows that the attraction leads to localization of pairs inside the noninteracting delocalized phase. This effect is absent in the mean-field approximation where the ground state remains well delocalized [compare Figs. 1(e) and 1(g)]. This phenomenon is

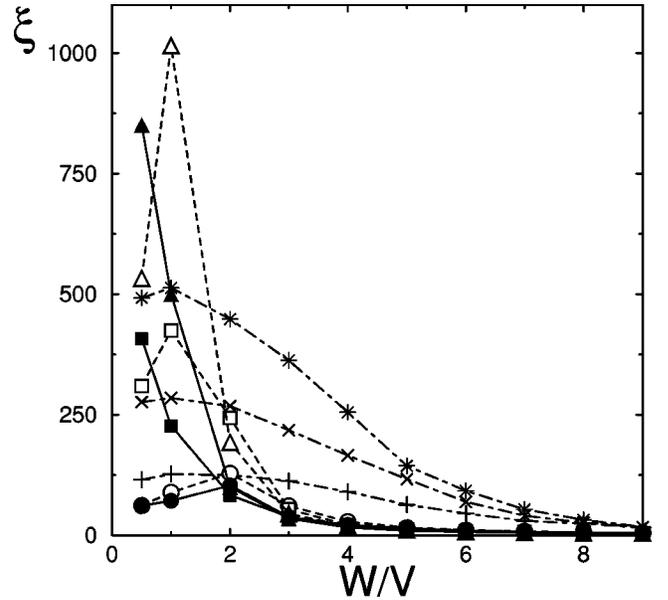


FIG. 2. Dependence of the inverse participation ratio  $\xi$  of the TIP ground state on the disorder  $W/V$ . The full lines with full symbols correspond to an interaction strength  $U=-4V$ , the dashed ones with open symbols to  $U=-2V$ , and the dot-dashed ones to  $U=0V$  (+,  $\times$ , \*). Different symbols correspond to different linear sizes of the lattice  $L=20$  ( $\circ$ , +),  $L=30$  ( $\square$ ,  $\times$ ), and  $L=40$  ( $\triangle$ , \*).

similar to the situation in the three-dimensional Anderson model for which the localization of pairs was discussed in Ref. 17. As in Ref. 17 we attribute this phenomenon to the increase of the effective mass  $m_{\text{eff}}$  of the pair ( $V \propto 1/m_{\text{eff}}$ ) that leads to a decrease of the critical disorder strength ( $W_c \sim V \propto 1/m_{\text{eff}}$ ). For strong attraction the mass is approximately doubled so that the value of  $W_c$  is decreased by a factor of 2 compared to the noninteracting case. The numerical data in two and three dimensions presented here and in Ref. 17 are in satisfactory agreement with this estimate.

The dependence of IPR  $\xi$  on the system size  $L$  for  $W=2V$  and  $W=5V$  is shown in Fig. 3 for different values of interaction  $U$ . For  $L \leq 12$  the noninteracting states are delocalized. The introduction of interaction decreases significantly the IPR value and for  $W=5V$  the TIP ground state is localized for  $|U| \geq 2V$ . On the contrary for  $W=2V$  the IPR still grows with  $L$  for  $U=-2V$ . This behavior is qualitatively similar to the numerical data obtained in Ref. 18 by the projected quantum Monte Carlo method (see Fig. 3 of Ref. 18). According to Ref. 18 the pairs at quarter filling become localized at  $U/V \approx -4$  for  $W=5V$  and remain delocalized for  $W=2V$  (only sizes  $L \leq 12$  were accessible by this method). While the qualitative behavior is similar (compare Fig. 3 with Fig. 3 in Ref. 18) the quantitative difference between the two sets of data is definitely present. For example in our Fig. 3 at  $W=5V$  the states become localized approximately at  $U/V \approx -1.5$  and not at  $U/V \approx -4$  as in Ref. 18. We attribute this quantitative difference to the fact that in Ref. 18 up to 74 real spin fermions were present and were treated exactly (up to statistical errors) by the quantum Monte Carlo method. The presence of other fermions can

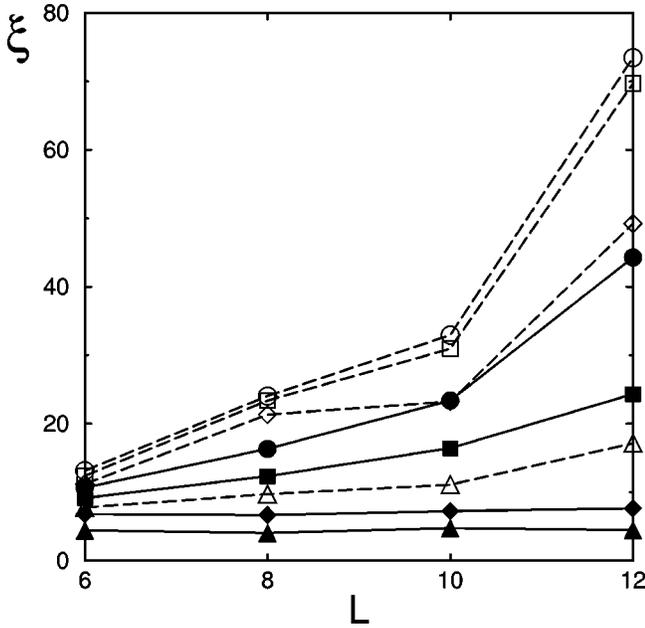


FIG. 3. IPR  $\xi$  as a function of the linear size  $L$ . Dashed lines and open symbols are for  $W/V=2$  and full lines and full symbols are for  $W/V=5V$ , with  $U/V=0$  ( $\circ$ ),  $-1$  ( $\square$ ),  $-2$  (diamond), and  $-4$  ( $\triangle$ ). The average is done over 100 disorder realizations. Here, the filling factor is  $\nu=\frac{1}{4}$  and  $\alpha=L^2/M=2$ .

renormalize the effective strength of interaction between two particles. Also it can change the effective strength of disorder for fermions near the Fermi level. The comparison between the two figures shows that the TIP approach captures the qualitative physical properties of the system but quantitatively it gives different values. In a sense this situation is similar to the comparison between the Cooper approximation and the BCS theory.

The difference between exact diagonalization of the TIP Hamiltonian (2) and the mean-field solution given by the Cooper ansatz is also clearly seen in the coupling energy of the pair  $\Delta = E_g(U=0) - E_g(U)$ . Here  $E_g$  is the TIP ground-state energy in the presence of interaction  $U$ . For  $U=0$  we have  $E_g(U=0) = 2E_F$ . The dependence of  $\Delta$  on the disorder strength  $W$  is shown in Fig. 4 for  $U=-2V$ . In the BLS phase at  $W > W_c \approx 3V$  the coupling energy  $\Delta$  obtained from exact diagonalization becomes significantly larger than the value of  $\Delta$  given by the mean-field approximation based on delocalized states. This shows that energetically the BLS phase is more favorable than the mean-field Bogolubov-de Gennes solution.<sup>24</sup> The physical reason for the increase of  $\Delta$  compared to the mean-field value is related to localization: the pairs are localized and particles remains closer to each other, which effectively increases the coupling strength between them.<sup>25</sup> On the contrary for  $W < W_c$  when the pairs are delocalized the exact solution gives the values of  $\Delta$ , which are close to the mean-field value. This is in agreement with the Anderson theorem according to which the mean field remains valid in the regime with weak disorder.

The fact that for TIP the BLS phase is energetically more favorable than the mean-field solution indicates that also at finite particle density the BLS phase will be more favorable.

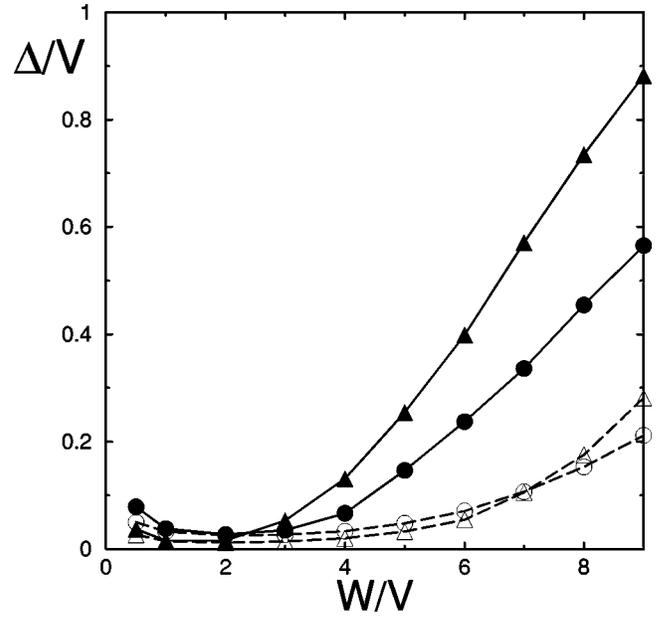


FIG. 4. Dependence of the coupling energy  $\Delta$  on the disorder strength  $W$  for  $U=-2V$ . The linear lattice size is  $L=20$  ( $\circ$ ) and  $L=40$  ( $\triangle$ ); data are obtained by exact diagonalization of the TIP model (solid lines, full symbols) and by mean-field approximation from the Cooper ansatz (dashed lines, empty symbols).

This indication is in agreement with the quantum Monte Carlo computations presented in Ref. 18. It is also possible to give another argument in favor of the BLS phase. Suppose that the filling factor  $\nu$  is close to the critical value  $\nu_c$  at the mobility edge of noninteracting particles (but  $\nu > \nu_c$ ). Then it is natural that all particles below the mobility edge are localized. Then the density of interacting pairs above  $\nu_c$  is proportional to  $|\nu - \nu_c|$  and is relatively low for  $|\nu - \nu_c| \ll \nu$ . In this regime the pairs above  $\nu_c$  are well separated and the TIP approximation we discuss in this paper should be rather reasonable. Of course in the next step the residual interaction between pairs should be taken into account.<sup>26</sup> In this picture it is clear that the BLS phase is energetically more favorable compared to the delocalized mean-field solution.

### III. GROUND-STATE PROPERTIES WITH MAGNETIC FIELD

It is interesting to understand how the TIP properties in the BLS phase are affected by a magnetic field  $\mathbf{B}$  perpendicular to the two-dimensional lattice. In this case the one-particle Hamiltonian takes the form

$$H_1 = \sum_{\mathbf{n}} E_{\mathbf{n}} |\mathbf{n}\rangle \langle \mathbf{n}| + V \sum_{\mathbf{n}} [T(\mathbf{n}) + T^*(\mathbf{n})], \quad (3)$$

where  $T(\mathbf{n})$  and  $T^*(\mathbf{n})$  are the translation operators from site  $\mathbf{n}$  to its nearest neighbors

$$T(\mathbf{n}) = T_{M_x}(\mathbf{n}) |\mathbf{n}\rangle \langle \mathbf{n} + \mathbf{e}_x| + T_{M_y}(\mathbf{n}) |\mathbf{n}\rangle \langle \mathbf{n} + \mathbf{e}_y|. \quad (4)$$

Here  $\mathbf{e}_x$  and  $\mathbf{e}_y$  are the unitary vectors on the two-dimensional lattice and

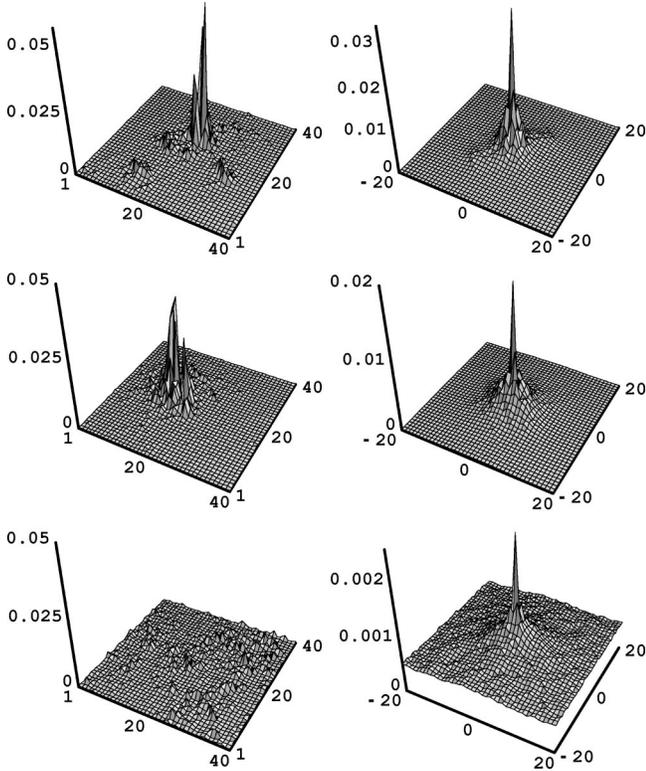


FIG. 5. Ground-state probability distributions of two interacting particles in a lattice of linear size  $L=40$ , with disorder strength  $W=3V$  and interaction strength  $U=-2V$ ; the left column shows the probability distribution  $f(\mathbf{n})$  and the right one the interparticle probability distribution  $f_d(\mathbf{r})$ . The different rows present the TIP ground state for different values of the magnetic flux  $\gamma$ : top row  $\gamma=0$ , middle row  $\gamma=2/40$ , and bottom row  $\gamma=5/40$ .

$$T_{M_{x(y)}}(\mathbf{n}) = \exp\left(-\frac{iq}{\hbar c} \int_{\Gamma_{x(y)}(\mathbf{n})} \mathbf{A} \cdot d\mathbf{n}'\right) \quad (5)$$

are the magnetic translation operators along paths  $\Gamma_x(\mathbf{n}) = (\mathbf{n} \rightarrow \mathbf{n} + \mathbf{e}_x)$  and  $\Gamma_y(\mathbf{n}) = (\mathbf{n} \rightarrow \mathbf{n} + \mathbf{e}_y)$ . For convenience we choose the Landau's gauge for the magnetic field  $\mathbf{A} = -n_y B \mathbf{e}_x$ . The magnetic translation operators are then determined as  $T_{M_x} = \exp(2\pi i \gamma n_y)$  and  $T_{M_y} = 1$  with  $\gamma = qB/\hbar c$ . Due to the periodic boundary conditions the effective topology of the two-dimensional lattice is that of a torus with a transversal and a longitudinal radius  $R_t = R_l = L$ . This topology implies flux quantization on the lattice so that  $\gamma = m/L$  with  $m \in [0, L-1]$ . Then the one-particle Hamiltonian (3) can be explicitly written as

$$H_1 = \sum_{\mathbf{n}} E_{\mathbf{n}} |\mathbf{n}\rangle \langle \mathbf{n}| + \sum_{\mathbf{n}} (e^{2\pi i \gamma n_y} |\mathbf{n}\rangle \langle \mathbf{n} + \mathbf{e}_x| + |\mathbf{n}\rangle \langle \mathbf{n} + \mathbf{e}_y|) + \sum_{\mathbf{n}} (e^{-2\pi i \gamma n_y} |\mathbf{n} + \mathbf{e}_x\rangle \langle \mathbf{n}| + |\mathbf{n} + \mathbf{e}_y\rangle \langle \mathbf{n}|). \quad (6)$$

We study now the ground state of the TIP Hamiltonian constructed with this new one-particle Hamiltonian  $H_1$  (6). This Hamiltonian is written in the preferential basis of non-interacting eigenstates of Eq. (6), which leads to the Schrödinger

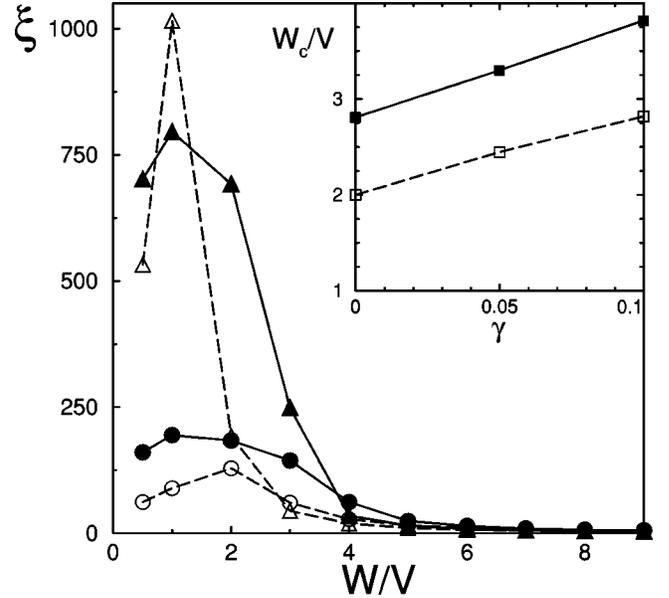


FIG. 6. Dependence of the inverse participating ratio  $\xi$  of the TIP ground state on the disorder  $W/V$  and magnetic flux  $\gamma$ . The interaction strength for the main figure is  $U=-2V$ . Empty/full symbols correspond to  $\gamma=0/\gamma=0.1$ , with different linear lattice sizes  $L=20$  ( $\circ$ ) and  $L=40$  ( $\triangle$ ). The inset shows the phase diagram in the plane of  $W_c/V$  and  $\gamma$ ; the full line corresponds to  $U=-2V$  and the dashed line to  $U=-4V$ ; the BLS phase is at  $W > W_c$ .

equation of the form (2). As in the case with  $\mathbf{B}=\mathbf{0}$  we use the probability distributions  $f(\mathbf{n})$ ,  $f_d(\mathbf{r})$ , and the IPR  $\xi$  to depict the ground-state properties of the TIP problem in the presence of a magnetic field. Thus Fig. 5 represents the TIP probability distributions for a system of linear size  $L=40$  and fixed disorder and interaction strengths ( $W=3V, U=-2V$ ). The data are shown for different magnetic-flux ratios  $\gamma=0, \gamma=2/40$ , and  $\gamma=5/40$ . At  $\gamma=0$  the one-particle probability is well localized by interaction (first row of Fig. 5). However, with the increase of magnetic flux  $\gamma$  the localization is destroyed. The data of Fig. 5 suggest that there exists a critical magnetic flux  $\gamma_c$  below which the TIP pairs remain localized ( $\gamma < \gamma_c$ , middle row of Fig. 5) and above which pairs become totally delocalized ( $\gamma > \gamma_c$ , bottom row of Fig. 5). At the same time for  $\gamma > \gamma_c$  the interparticle distance probability distribution  $f_d(\mathbf{r})$  is less peaked. Hence for  $\gamma > \gamma_c$  the size of the pair is significantly increased.

The ground-state properties can be studied in a more quantitative way with the help of the IPR  $\xi$  defined above. Figure 6 represents the dependence of  $\xi$  on the disorder strength  $W$  for a fixed interaction  $U=-2V$ . Data are shown for different values of magnetic flux  $\gamma$ . They clearly show that the introduction of magnetic field leads to an increase of  $\xi$  at a fixed value of  $W$ . Thus the magnetic field enhances the delocalization of particles for  $\gamma > \gamma_c(U)$  and  $W < W_c(U)$ . On the contrary for  $\gamma < \gamma_c(U)$  and  $W > W_c(U)$  the variation of  $\xi$  with an lattice size  $L$  is weak and usually here there is a small decrease of  $\xi$  with an increase of  $L$  (see Fig. 6). The delocalization transition can be determined as the point where  $\xi$  is independent of the lattice size (crossing point). An

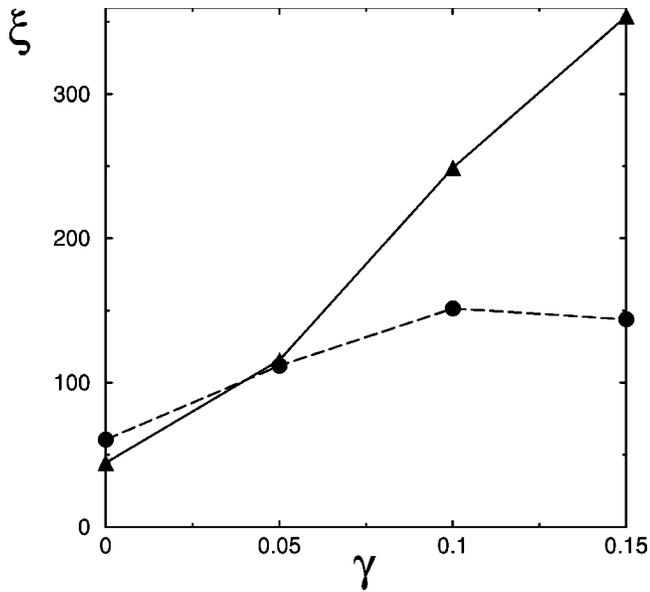


FIG. 7. Dependence of the inverse participation ratio  $\xi$  of the TIP ground state on the magnetic flux  $\gamma$  for two lattice sizes  $L=20$  (●) and  $L=40$  (triangle). Here the interaction strength is  $U=-2V$  and the disorder strength is  $W=3V$ .

approximate phase diagram in the plane  $(W, \gamma)$  obtained in this way is shown in the inset of Fig. 6. With the increase of interaction the localized phase penetrates deeper in the region of weak disorder. The fact that a magnetic field can delocalize pairs inside the BLS phase is also illustrated in Fig. 7 where the IPR  $\xi$  is enormously increased by the magnetic flux. This result is in agreement with a general fact known for noninteracting particles that the localization length is increased by a magnetic field.<sup>19</sup> While the delocalization induced by a magnetic field is clearly illustrated by Figs. 6 and 7 it is rather difficult to determine numerically the properties of pairs in the delocalized phase. In this phase an effective pair size becomes too large to investigate numerically. The question of whether the superconductivity survives or if a magnetic field drives the system to a metallic regime is difficult to answer in the frame of our numerical approach.

For a better understanding of both localized and delocalized phases we studied the dependence of the pair coupling energy  $\Delta$  on the strength of the magnetic field. In the standard Cooper problem  $\Delta$  is related to the BCS gap and determines the Cooper pair size  $l_{pair} \propto 1/\Delta$ . Figure 8 shows the dependence of  $\Delta$  on the magnetic field for different lattice sizes and for the case  $U=-2V$  and  $W=3V$ , already used in Fig. 7. In the localized regime with  $\gamma < \gamma_c \approx 0.04$  the value of  $\Delta$  varies weakly with the growth of  $L$ . In contrast, for  $\gamma > \gamma_c$  its value decreases in two to three times. This can be

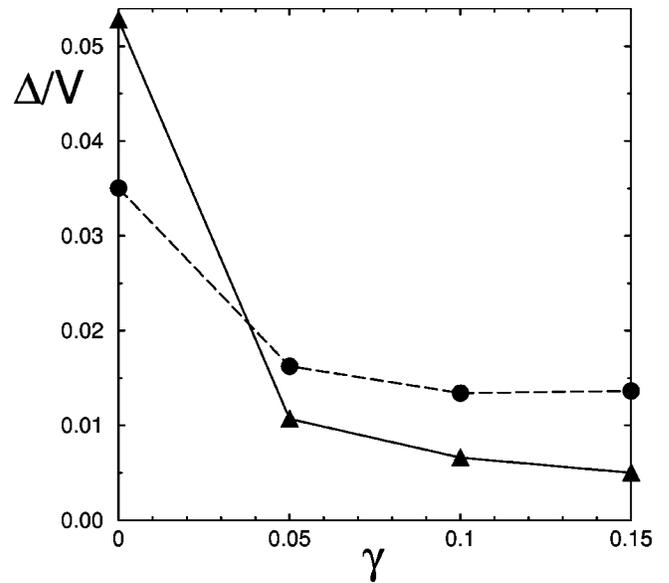


FIG. 8. Pair coupling energy  $\Delta$  as a function of magnetic flux  $\gamma$  for  $L=20$  (●) and  $L=40$  (triangle). The interaction and disorder strengths are as in Fig. 7:  $U=-2V$  and  $W=3V$ .

considered an indication that the superconductivity is significantly suppressed by magnetic field for  $\gamma > \gamma_c$ . However, a significant increase of  $L$  is required to investigate the properties of pairs in the delocalized regime.

#### IV. CONCLUSION

The present studies show that in the presence of disorder the attractive Hubbard interaction leads to localization of pairs and the appearance of a phase with biparticle localized states that is located inside the noninteracting metallic regime. This BLS phase cannot be obtained in the mean-field approximation. It is shown that it can be destroyed by the introduction of a magnetic field, which drives the system to delocalization. In the BLS phase the pair coupling energy is much larger than the value obtained in the mean-field approximation. This indicates that the BLS phase is energetically more preferable compared to the mean-field solution. The results obtained for two particles (one pair) are in qualitative agreement with the recent results obtained with the quantum Monte Carlo method in Ref. 18.

#### ACKNOWLEDGMENTS

It is our pleasure to thank G. Benenti and B. Srinivasan for stimulating discussions. We thank the IDRIS in Orsay and the CalMiP in Toulouse for access to their supercomputers.

\*Email address: <http://w3-phystheo.ups-tlse.fr/~dima>

<sup>1</sup>A. Larkin, Ann. Phys. (Leipzig) **8**, 785 (1999).

<sup>2</sup>K. A. Matveev and A. I. Larkin, Phys. Rev. Lett. **78**, 3749 (1997).

<sup>3</sup>M. Wallin, E. S. Sørensen, S. M. Girvin, and A. P. Young, Phys. Rev. B **49**, 12 115 (1994).

<sup>4</sup>R. T. Scalettar, N. Trivedi, and C. Huscroft, Phys. Rev. B **59**, 4364 (1999).

<sup>5</sup>M. Franz, C. Kallin, A. J. Berlinsky, and M. I. Salkola, Phys. Rev. B **56**, 7882 (1997).

<sup>6</sup>A. Ghosal, M. Randeria, and N. Trivedi, Phys. Rev. B **63**,

- 020505(R) (2001).
- <sup>7</sup>P. J. H. Denteneer, R. T. Scalettar, and N. Trivedi, Phys. Rev. Lett. **83**, 4610 (1999).
- <sup>8</sup>A. M. Goldman and N. Marković, Phys. Today **51** (11), 39 (1998).
- <sup>9</sup>V. F. Gantmakher, M. V. Golubkov, V. T. Dolgoplov, G. E. Tsydnyzhapov, and A. A. Shashkin, Pis'ma Zh. Eksp. Teor. Fiz. **68**, 337 (1998) [JETP Lett. **68**, 363 (1998)].
- <sup>10</sup>G. S. Boebinger, Y. Ando, A. Passner, T. Kimura, M. Okuya, J. Shimoyama, K. Kishio, K. Tamasaku, N. Ichikawa, and S. Uchida, Phys. Rev. Lett. **77**, 5417 (1996).
- <sup>11</sup>K. Segawa and Y. Ando, J. Low Temp. Phys. **117**, 1175 (1999).
- <sup>12</sup>P. W. Anderson, J. Phys. Chem. Solids **11**, 26 (1959).
- <sup>13</sup>A. A. Abrikosov and L. P. Gorkov, Zh. Éksp. Teor. Fiz. **35**, 1558 (1959) [Sov. Phys. JETP **8**, 1090 (1959)].
- <sup>14</sup>G. Montambaux, D. Poilblanc, J. Bellissard, and C. Sire, Phys. Rev. Lett. **70**, 497 (1993).
- <sup>15</sup>E. Dagotto, Rev. Mod. Phys. **66**, 763 (1994).
- <sup>16</sup>L. N. Cooper, Phys. Rev. **104**, 1189 (1956).
- <sup>17</sup>J. Lages and D. L. Shepelyansky, Phys. Rev. B **62**, 8665 (2000).
- <sup>18</sup>B. Srinivasan and D. L. Shepelyansky, cond-mat/0102055 (unpublished).
- <sup>19</sup>A. Mirlin, Phys. Rep. **326**, 259 (2000).
- <sup>20</sup>Y. Imry, Europhys. Lett. **30**, 405 (1995).
- <sup>21</sup>Ph. Jacquod and D. L. Shepelyansky, Phys. Rev. Lett. **78**, 4986 (1997).
- <sup>22</sup>F. von Oppen, T. Wettig, and J. Müller, Phys. Rev. Lett. **76**, 491 (1996); *Correlated Fermions and Transport in Mesoscopic Systems*, edited by T. Martin, G. Montambaux, and J. Trân Thanh Vân (Editions Frontieres, Gif-sur-Yvette, 1996) p. 235.
- <sup>23</sup>M. A. N. Araújo, M. Dzierzawa, C. Aebischer, and D. Baeriswyl, Physica B **244**, 9 (1998).
- <sup>24</sup>P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966).
- <sup>25</sup>In the BLS phase at  $W > 3V$  the value of  $\Delta$  obtained by exact diagonalization (full symbols) demonstrates some increase with the change of  $L$  from  $L=20$  to  $L=40$ . We attribute this to the fact that at larger  $L$  the pair in the ground state can choose a more profound minimum appearing due to statistical fluctuations that gives some growth of  $\Delta$ .
- <sup>26</sup>The first step in direct numerical investigations of a few interacting spin fermions above the frozen Fermi sea was done in J. Lages, G. Benenti, and D.L. Shepelyansky, Phys. Rev. B **63**, 214516 (2001). However in this approach it is difficult to reach large system sizes since the condition  $M/L^2 = \text{const}$  leads to a strong growth of the matrix size with the number of particles.