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Superinsulator as a phase of bi-particle localized states

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Abstract. We propose a physical picture of superinsulator observed recently in experiments with superconducting films in a magnetic field. On the basis of previous numerical studies we argue that a moderate attraction creates bi-particle localized states at intermediate disorder strength when noninteracting electron states are delocalized and metallic. Our present numerical study show that such localized pairs are broken by a static electric field which strength is above a certain threshold. We argue that such a breaking of localized pairs by a static field is at the origin of superinsulator breaking with a current jump observed experimentally above a certain critical voltage.

1 Introduction

An interplay of disorder, Anderson localization and superconductivity attracts active experimental and theoretical interest (see e.g. reviews [1,2] and a recent research article [3]). A weak disorder does not significantly affect the superconducting phase in agreement with the Anderson theorem [4,5]. However, a relatively strong disorder can lead to a nontrivial situation when an attraction between electrons creates bi-particle localized states (BLS phase) from noninteracting metallic delocalized electron states [6,7] (see Figs. 1a, 1b). First signatures of this BLS phase has been obtained in the frame of the generalized Cooper problem [8] of two interacting particles above a frozen Fermi sea in presence of disorder. Further extensive quantum Monte Carlo studies of two and three-dimensional (2D, 3D) Hubbard model [9,10], with attraction, disorder and about hundred electrons, confirmed the attraction induced picture of localized Cooper pairs proposed in [6,7]. We note, however, that the BLS picture is not very conventional for usual solid-state community views which are more centered on renormalization group with further developments of approach proposed in [11]. More recent theoretical approaches can be find in [3,12] and references therein.

The experimental studies of superconductor-insulator transition (SIT) in presence of disorder were started with InO_x alloys [13–16] and later continued with other materials (see [1,2] and references therein). The picture of Cooper pairs localized by disorder was especially emphasized on the basis of 2D disordered films experiments on SIT by Gantmakher et al. [16]. These experiments clearly display a presence of localized pairs in the insulating phase

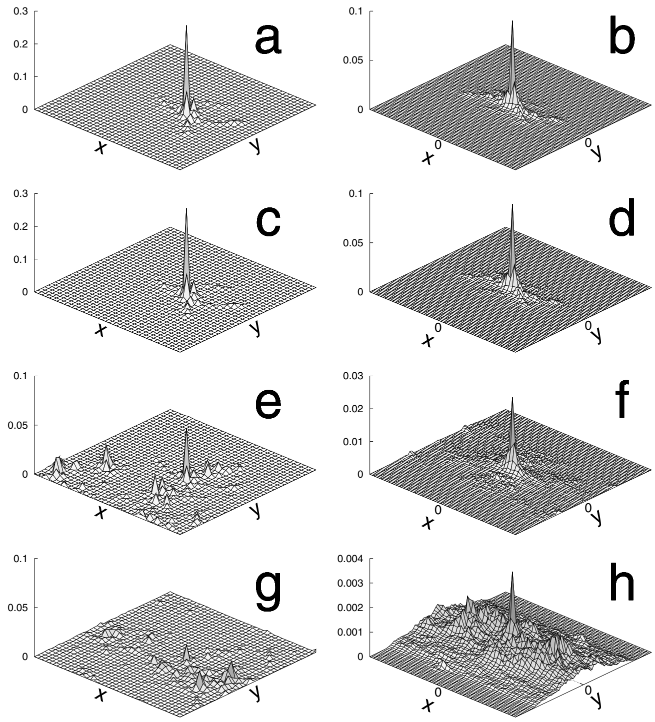

Fig. 1. Delocalization of two interacting particles with attractive Hubbard interaction $U = -2V$ by a static electric force F. The generalized Cooper problem is considered on 2D lattice of size $L \times L = 40 \times 40$ at disorder strength $W = 5V$, a static field F is directed along y-axis. Probability is shown for a lowest energy eigenstate with a maximum probability $f(y) = \sum_x f(x, y)$ at $y = L/2$. Left panels show one-particle probability $f(x, y)$ and right panels show interparticle distance probability $f_d(x, y)$. The static electric force, directed along y-axis, is $F = 0(a, b)$, $F = 0.003V(c, d)$, $F = 0.016V(e, f)$, $F = 0.052V(g, h)$.

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appearing from the superconducting phase at relatively large magnetic field (see also [2] and references therein). At even larger magnetic fields these samples show a metallic type behavior which is argued to correspond to underlying metallic noninteracting states and the BLS phase [7].

Recent experiments also discovered that in 2D disordered films the above insulating phase is abruptly destroyed by a static *dc*-voltage [17–21]. It was argued that this unusual insulating phase is related to a certain collective state named *superinsulator* [20]. The physical origin of this state is under hot theoretical debates [20,22–26]. However, this physical problem involves nontrivial interplay of interactions, disorder and localization which is not easy to handle by purely analytical methods. Indeed, it is known that repulsive or attractive interactions can produce delocalization of two interacting particles above Fermi level when all one-particle states are exponentially localized due to the Anderson localization (see e.g. [27–32]). This two interacting particles (TIP) effect leads to an effective 3D Anderson transition for TIP excitations at certain energy above the Fermi level in the case of Coulomb or other long range interactions [33,35]. But at the same time attractive interactions create the BLS phase in the vicinity of Fermi level even when noninteracting states are metallic and delocalized [6,7,9,10]. Due to that for a better understanding of physics of superinsulator it would be rather useful to study numerically the effects of a static electric field on localization-delocalization transition in presence of interactions. A certain progress has been reached in studies of attractive Hubbard model with disorder by quantum Monte Carlo methods since there is no sign problem in such a case and numerical simulations can be done with a large number of electrons (see e.g. [9,10,36] and references therein). However, this approach is not easy to adapt to a case of finite static field.

Even if the quantum Monte Carlo studies [9,10] of many-electron problem confirm in general the picture of BLS pairs obtained in the frame of the generalized Cooper problem of TIP [6,7] a detailed analysis of this phase is still at its initial stage. A number of questions remain to be understood including collective properties of localized pairs, the phase coherence between various localized regions through the whole system, possibility of local order parameter and global correlation properties. A similarity between BLS pairs, found for the generalized Cooper problem and the quantum Monte Carlo methods, indicates that there is no strong correlations between BLS pairs in various localized regions but detailed investigation is lacking. The main reason is that the interacting many-body problem is rather difficult for numerical studies and at the same time analytical methods are not adapted to this complex problem (e.g. the Bogolubov – de Gennes mean field approach does not capture the BLS phase [9,10]).

In this work we study effects of static field in the frame of the generalized Cooper problem using the approach of two interacting particles with an attractive Hubbard interaction U [6,7]. Qualitative description of a physical picture of BLS phase and its destruction by a static electric field F is given in Section 2. In Section 3 we describe our model of the generalized Cooper problem in presence of *dc*-field, the results of numerical simulations are presented in Section 4, discussion of physical results and comparison with experiments are given in Section 5.

2 Physical picture of superinsulator

The results obtained in [6,7,9,10] give the following qualitative physical picture of the BLS phase, which is at the origin of superinsulator as it is argued below:

(i) For a moderate disorder, which e.g. by a factor two or less smaller than the critical value for the Anderson transition for 3D case, noninteracting electron states are still delocalized and metallic. A similar situation appears for finite size 2D samples where one-particle localization length is larger than the sample size.

(ii) However, an attractive Hubbard interaction creates singlet spin pairs of two electrons which total mass is twice larger than electron mass, hence, an effective hopping matrix element of such a pair becomes twice smaller or, equivalently, effective strength of disorder becomes twice larger that leads to localization of pairs and BLS phase. Such localized pairs have a certain localization size ℓ and a coupling energy Δ. It is important to stress that the Bogoliubov – de Gennes meanfield approach is not able to capture this BLS phase [6,7,9,10]. The BLS phase is an insulator existing at temperature $T < \Delta$. In a certain sense attraction between two electrons creates an effective well which captures localized electron pairs. However, excitations with energy $\Delta E > E_D \approx \Delta$ are delocalized. Indeed, even if all one-particle states are localized, the excited states above the Fermi level become delocalized by interactions between electrons as it is shown in [30,32,33,35]. Moreover, the energy excitations of BLS phase are delocalized above certain energy $E_D \approx \Delta$ since noninteracting states are metallic. According to the Fermi-Dirac statistical distribution an excitation probability to high energy drops exponentially and therefore for $T < \Delta$ the resistivity R_{xx} is characterized by the Arrhenius law $R_{xx} \propto \exp(T_0/T)$ with $T_0 \approx E_D \approx \Delta$.

(iii) In real 2D superconducting films, studied experimentally (see e.g. [2,16]), an increase of magnetic field B up to a few Tesla effectively decreases attraction inside Cooper pairs due to breaking of time reversal and also effectively increases an effective strength of the disorder since magnetic field increases a return probability and scattering on impurities. As a result superconductivity disappears, electron pairs become localized, but at even stronger magnetic field attraction between electrons is completely eliminated and one obtains a metal of noninteracting electrons. This effective change of attraction U and disorder strength W with an increase of magnetic field B is schematically shown in Figure 2 by a dashed curve. In this picture, resistivity R_{xx} initially grows with increase of B but above a certain value it starts to decrease with B giving a peak in R_{xx}, which is a characteristic feature of experiments (see e.g. Fig. 1 in [16]).

(iv) Thus, the BLS phase and its energy excited states can be viewed as a sequence of wells with localized pairs

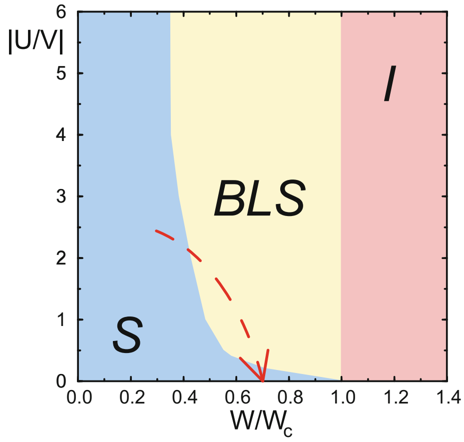

Fig. 2. (Color online) Phase diagram in the 3D Anderson model with disorder strength W and Hubbard attraction U: superconductor (S), phase of localized pairs (BLS), Anderson insulator (I); $W_c \approx 16.5V$ is the critical disorder for noninteracting particles at half filling. The data, taken from [6], are obtained in the frame of the generalized Cooper problem with TIP. The dashed curve with arrow schematically shows variation of effective U and W values with the increase of magnetic field corresponding to the experiment [16] (see Sect. 2 for more details). This phase diagram is shown for zero temperature T.

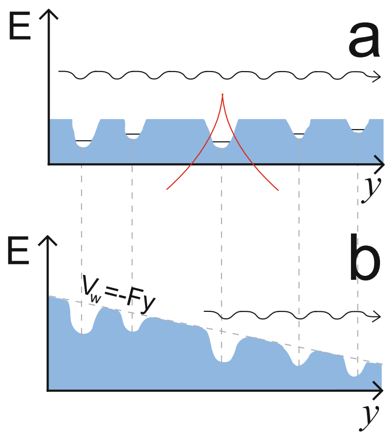

Fig. 3. (Color online) Pictorial image of the energy spectrum of BLS pairs localized by a Hubbard attraction and located at various positions along y-direction of the lattice at static force $F = 0$ (a); delocalization of BLS pairs by a washboard potential $V_w = -Fy$ of static field at $F > F_c$ (b).

and continuum of delocalized states as it is schematically shown in Figure 3a. A static electric field with force F creates an energy slope leading to a tilted washboard potential as it is schematically shown in Figure 3b. Above a certain critical force $F > F_c$ the localized states inside a well (localized pairs) become coupled to continuum states of delocalized electrons that creates an avalanche of delocalized electrons. For $F > F_c$ all electrons become delocalized that produces a sharp increase (jump) of electron current which is a characteristic feature of superinsulator experiments [17–21]. The critical field F_c above which the superinsulator is destroyed can be estimated by taking into account that the coupling energy of localized pairs Δ should be comparable with the energy change in a static field F_c on the pair size ℓ that gives

$$F_c \approx \Delta/\ell, \quad V_c = F_c L. \tag{1}$$

The physical meaning of this relation is rather direct: a strong field breaks BLS pairs and creates a charge current. A critical voltage V_c, at which a jump of current takes place in an experiment, is proportional to the sample size L.

In next sections we present numerical simulations of the generalized Cooper problem in a tilted potential which justifies this physical picture.

3 Model description

For our numerical studied of delocalization of BLS pairs by a static electric field F we use 2D Anderson model with disorder strength W and Hubbard attraction U between two particles. Following [7], with the same notations, we use the one-particle Hamiltonian:

$$H_1 = \sum_{\mathbf{n}} (E_{\mathbf{n}} + \mathbf{F}\cdot\mathbf{n})\,|\mathbf{n}\rangle\langle\mathbf{n}| + V \sum_{\langle \mathbf{n},\mathbf{n}'\rangle} |\mathbf{n}\rangle\langle\mathbf{n}'| \tag{2}$$

where $\mathbf{n}$ and $\mathbf{n}'$ are index vectors on the two-dimensional square lattice with periodic boundary conditions in x-direction, and zero boundary conditions in y-direction, V is the nearest neighbor hopping term and the random on-site energies $E_{\mathbf{n}}$ are homogeneously distributed in the energy interval $\left[-\frac{W}{2}, \frac{W}{2}\right]$. We choose the direction of a static electric force $\mathbf{F}$ along y-axis. We consider a square lattice with linear size L up to $L = 40$. At such sizes the eigenstates at a half filling $\nu = 1/2$ are practically delocalized over the whole lattice for $W < 7V$ and $F = 0$ so that such finite samples can be considered to be metallic (see more details in [7]). We consider the particles in the singlet state with zero total spin so that the spatial wavefunction is symmetric with respect to particle permutation (interaction is absent in the triplet state).

To take into account the effects of Hubbard interaction we write the TIP Hamiltonian in the basis of noninteracting eigenstates at $F = 0$:

$$\begin{aligned}(E_{m_1} + E_{m_2})\chi_{m_1,m_2} &+ \sum_{m'} (F_{m_1,m'}\chi_{m',m_2} \\ &+ F_{m_2,m'}\chi_{m_1,m'}) + U \sum_{m_1',m_2'} Q_{m_1,m_2,m_1',m_2'}\chi_{m_1',m_2'} \\ &= E\chi_{m_1,m_2}.\end{aligned} \tag{3}$$

Here E_m are the one-particle eigenenergies corresponding to the one-particle eigenstates $|\phi_m\rangle$ and χ_{m_1,m_2} are the components of the TIP eigenstate in the noninteracting eigenbasis $|\phi_{m_1}, \phi_{m_2}\rangle$ at $F = 0$. The matrix elements $F_{m_1,m'}$ describe the static force transitions between one-particle eigenstates $|\phi_{m_1}, \phi_{m_2}\rangle$. The matrix elements $UQ_{m_1,m_2,m_1',m_2'}$ give the interaction induced transitions between non-interactive eigenstates $|\phi_{m_1},\phi_{m_2}\rangle$ and $|\phi_{m_1'},\phi_{m_2'}\rangle$. These matrix elements are obtained by rewriting the Hubbard interaction in the non-interactive eigenbasis of model (2) at $F=0$. In the analogy with the original Cooper problem [8] the summation in (3) is done over the states above the Fermi level with eigenenergies $E_{m_{1,2}'} > E_F$ with $m_{1,2}' > 0$. The Fermi energy $E_F \approx 0$ is determined by a fixed filling factor $\nu = 1/2$. To keep the similarity with the Cooper problem we restrict the summation on $m_{1,2}'$ by the condition $1 < m_1' + m_2' \le M$. In this way the cut-off with M unperturbed orbitals introduces an effective phonon energy $E_{ph} = \hbar\omega_D \approx 3.75VM/L^2 = 3.75V/\alpha$ where L is the linear system size. When varying L we keep $\alpha = L^2/M$ fixed so that the phonon energy is independent of system size. All the data in this work are obtained with $\alpha = 15$ but we also checked that the results are not sensitive to the change of α. We note that the Hamiltonian (3) exactly describes the noninteracting problem.

To analyze the effects of static force on localized BLS pairs we solved numerically the Schrödinger equation (3). After that we rewrite the obtained eigenstates in the original lattice basis with the help of the relation between lattice basis and one-particle eigenstates $|\mathbf{n}\rangle = \sum_m R_{\mathbf{n},m}|\phi_m\rangle$. As a result of this procedure we obtain the two-particle probability distribution $f_2(\mathbf{n_1},\mathbf{n_2})$ from which we extract the one-particle probability $f(\mathbf{n}) = \sum_{\mathbf{n_2}} f_2(\mathbf{n_1},\mathbf{n_2})$ and the probability of interparticle distance $f_d(\mathbf{r}) = \sum_{\mathbf{n_2}} f_2(\mathbf{r}+\mathbf{n_2},\mathbf{n_2})$ with $\mathbf{r} = \mathbf{n_1} - \mathbf{n_2}$. The localization properties are characterized by the one-particle inverse participation ratio (IPR) $\xi = \sum_{\mathbf{n}} f(\mathbf{n}) / \sum_{\mathbf{n}} f^2(\mathbf{n})$.

While in [7] only the ground state properties of given disorder realization have been studied here we also investigate the properties of excited states with the TIP energy ΔE counted from the Fermi level of noninteracting particles: $\Delta E = E - 2E_F$, where E is the eigenenergy of (3). We also consider only eigenstates with a maximum of one-particle probability inside the space range $-L/4 \le y \le L/4$ to avoid finite size effects in y-direction. In addition, we analyze only those eigenstates where the overlap probability of TIP to be on the same site is relatively large $f_d(0,0) > 5/L(2L-1)$. In this way the states with strongly separated particles are eliminated. Such an approach approximately corresponds to a finite particle density. We use usually $N_D = 30$ disorder realizations for statistical average.

Below we present numerical results for $U = -2V$, $W = 5V$ and $L \le 40$ at various values of F. The detailed studies presented in [7] ensure that at $F = 0$ these conditions are located well inside the BLS phase when the noninteracting states are delocalized (see Fig. 1b in [7]) while the ground state in presence of Hubbard attraction is well localized (see Figs. 1e, 1f in [7] and Figs. 1a, 1b here). We checked that the behavior in F remains similar at other values of parameters, e.g. $W = 3V$.

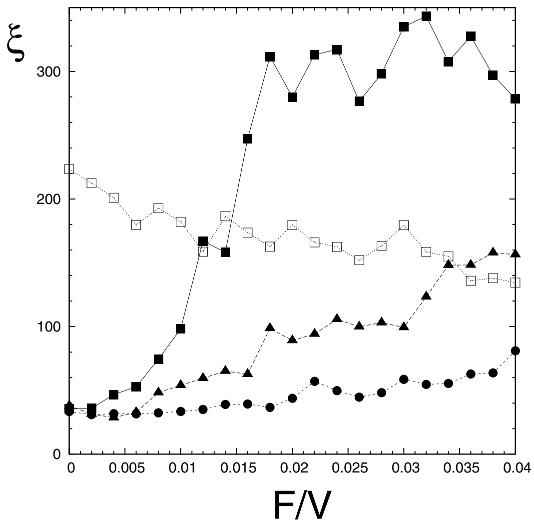

Fig. 4. Dependence of the average IPR ξ of lowest energy states on the static force F; average is done over $N_l = 15$ lowest energy states with a maximum of $f(x,y)$ inside a stripe $0.4L \le y \le 0.6L$ in a middle of the lattice at $y = L/2$ for $N_D = 30$ disorder realizations; the lattice size is $L = 20$ (●), $L = 30$ (▲), $L = 40$ (■) at $U = -2V$; the values of ξ for noninteracting case are shown by open symbols (□) for $L = 40, U = 0$. Here $W = 5V$.

4 Numerical results

The delocalization of BLS pairs by a static electric force F is illustrated in Figure 1 for one specific disorder realization. Here the disorder strength is relatively weak so that at $U = 0$ noninteracting eigenstates taken at half filling $\nu = 1/2$ and $E_F \approx 0$ are delocalized over the whole lattice of size $L = 40$ (see Fig. 1b in [7]). At $F = 0$ a moderate Hubbard attraction $U = -2V$ creates localized pairs with a certain coupling energy Δ (Figs. 1a, 1b). This localization remains robust against a weak static field (Figs. 1c, 1d) but at larger fields the localization is destroyed by a static force and particles become delocalized over the whole lattice (Figs. 1e–1h; to avoid boundary effects at finite F we select states in the middle of the lattice at $y = L/2$). The probability to have two particles close to each other also drops drastically for $F > F_c$ clearly demonstration pair breaking.

The dependence of average IPR ξ of lowest energy states on the static force F is shown in Figure 4. At small $F < F_c$ the values of ξ are size independent being much smaller compared to the case of noninteracting particles. This shows that a Hubbard attraction gives localization of pairs at low energy. For $F > F_c$ IPR starts to grow with the system size L demonstrating breaking of pairs and particle delocalization over the whole lattice. According to the data of Figure 4 we have $F_c \approx 0.015V$ at given $U = -2V$ and $W = 5V$. The data in Figure 5 give the coupling energy $\Delta \approx 0.1V$. Hence, this F_c value is in a good agreement with a simple estimate (1) with a numerical factor $A = F_c\ell/\Delta \approx 1$ corresponding to $\ell(F=0) \approx \sqrt{\xi(F=0)} \approx 6$ and $\Delta = 0.1V$.

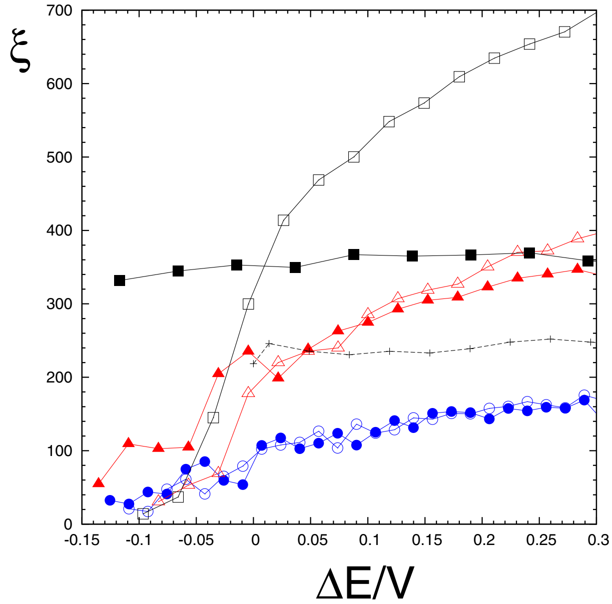

Fig. 5. (Color online) Average IPR ξ, for states peaked inside a stripe $0.4L \le y \le 0.6L$, as a function of the coupling energy $\Delta E = E - 2E_F$ for $U = -2V, W = 5V$ and lattice size $L = 20$ (blue circles), $L = 30$ (red triangles), $L = 40$ (black squares), at field $F = 0$ (open symbols) and $F = 0.036V$ (full symbols). The same averaged IPR ξ for $U = 0, W = 5V, F = 0$ and $L = 40$ is shown by (+) symbols.

The dependence of ξ on coupling energy $\Delta E = E - 2E_F$, for states in the middle of the lattice at $y \approx L/2$, is shown in Figure 5. According to this data the coupling energy is $\Delta \approx 0.1V$. This value is by a factor 2 smaller than the one found in [7] since only one lowest state for a given disorder realization was taken in [7] while here we average over few lowest states and also allow a relatively weak overlap f_d between TIP states. The states with $-\Delta \le \Delta E < 0$ are well localized at $F = 0$ since its IPR ξ is independent of lattice size L. In contrast, for $F = 0.036V > F_c$ the IPR ξ grows with the system size L showing that in this regime the states are delocalized.

At energies $\Delta E > 0$ the IPR grows with energy and becomes comparable with the system size L^2; also its is not sensitive to F. This is in agreement with the fact that noninteracting states are delocalized. Also interaction for excited states gives an additional TIP delocalization. For large values of $L = 40$ the static force gives a certain restriction of eigenstates spreading along the force direction due to the TIP energy conservation that gives a decrease of IPR value comparing to the case $F = 0$.

A more detailed dependence of ξ on coupling energy ΔE and static force F is given in Figure 6. For $U = 0$ we have $\xi \approx 200$ which is practically independent of ΔE and F while for $U = -2V$ we have very small $\xi \sim 10$ for $F = 0$ and large $\xi \sim 300$ for large F at $-0.1 < \Delta E/V < 0$. For energies $\Delta E > 0$ the states are delocalized at all F. These data also confirm the picture of field induced destruction of the BLS phase and delocalization.

Dependence of ξ on y and coupling energy ΔE is shown in Figure 7 for different values of F with and without in-

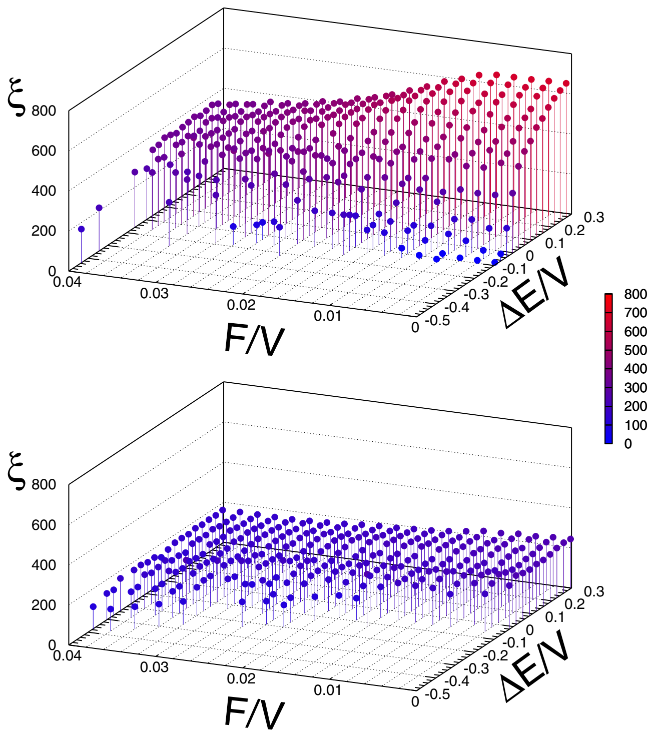

Fig. 6. (Color online) Average IPR ξ, for states peaked inside a stripe $0.4L \le y \le 0.6L$, as a function of the coupling energy $\Delta E = E - 2E_F$ and static force F for $U = -2V, W = 5V$ (top panel) and $U = 0, W = 5V$ (bottom panel); here $L = 40$.

teraction. For $U = 0$ we have ξ practically independent of y and ΔE in agreement with previous data of Figures 4–6. In contrast, in presence of attraction the states in the middle of the lattice in y have small ξ (localized) at lowest energies for small F (panels b, d) and have large ξ (delocalized) for large F (panels f, h). However, at the ends of the lattice in y direction the values of ξ are less sensitive to F due to boundary effects. Indeed, the static field forces particles to stay close to the boundary at y-ends of the lattice and hence the field induced delocalization is not well visible in this region. Due to that reason we use the states in the middle of the lattice to detect field induced delocalization in a clear way in Figures 4–6.

5 Discussion

The obtained numerical results confirm the physical picture of superinsulator destruction by a static field described in Section 2: the BLS pairs, localized by attraction inside noninteracting metallic phase, in presence of static field start to be coupled with higher energy delocalized states and above certain threshold $F > F_c$ (1) all pairs become delocalized. This creates an avalanche of delocalized electrons which gives an enormous increase of current in agreement with experimental observations. Of course, our numerical data detect delocalization of only one pair in a given disorder realization. Due to that in the frame of our studies we cannot obtain direct indications on resistivity variation by a few orders of magnitude as it is

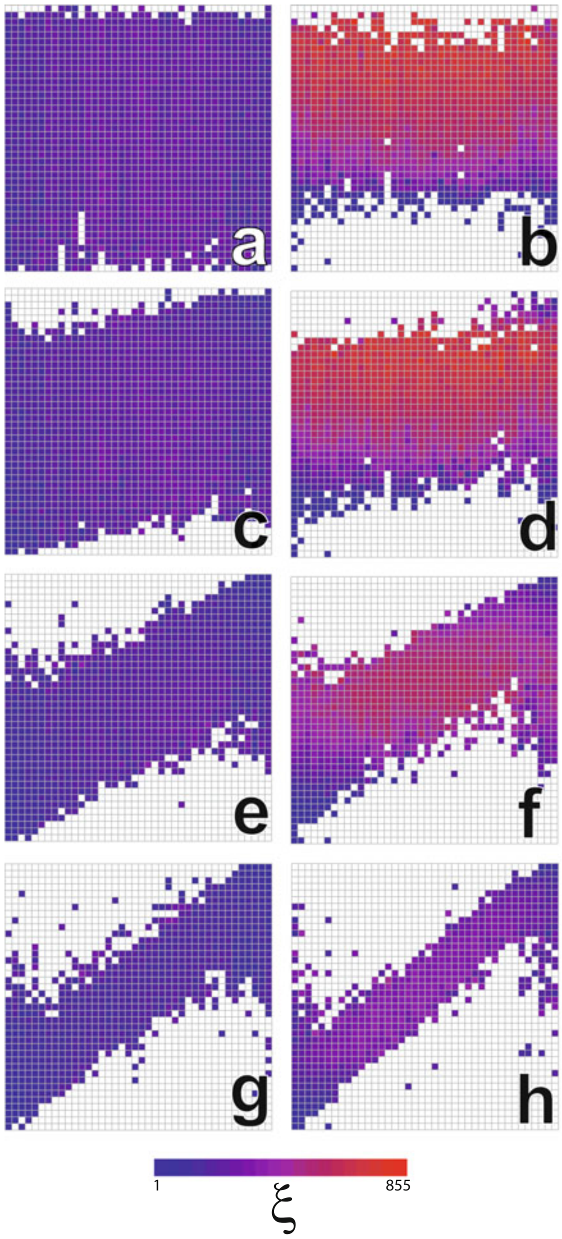

Fig. 7. (Color online) The IPR ξ for $W = 5V$ at $U = 0$ (left column) and $U = -2V$ (right column), at different values of the static force $F = 0$ (a, b), $F = 0.004V$ (c, d), $F = 0.016V$ (e, f), $F = 0.04V$ (g, h). Each panel has 40×40 cells, the vertical direction corresponds to the coupling energy $\Delta E = E - 2E_F$, the horizontal one to $1 \leq y \leq 40$. The bottom row corresponds to the lowest coupling energy, and the upper row corresponds to the highest coupling energy within the energy intervals at $U = 0$ being $(0, 0.54V)$ (a), $(-0.05V, 0.59V)$ (c), $(-0.25V,\ 0.78V)$ (e), $(-0.69V, 1.18V)$ (g) and at $U = -2V$ being $(-0.2V,\ 0.39V)$ (b), $(-0.23V,\ 0.57V)$ (d), $(-0.39V,\ 0.75V)$ (f), $(-0.77V,\ 1.17V)$ (h). The cell color gives the average ξ inside the cell (with ΔE being in the corresponding energy range defined by the row, and the maximum of probability distribution $f(y)$ along y, being located at y position defined by the column). For each panel $\sim$30 000 states are used with $N_d = 30$ disorder realizations. These states are selected in such a way that the probability of two particles located at the same site is greater than $5/L(2L-1)$; here $L = 40$.

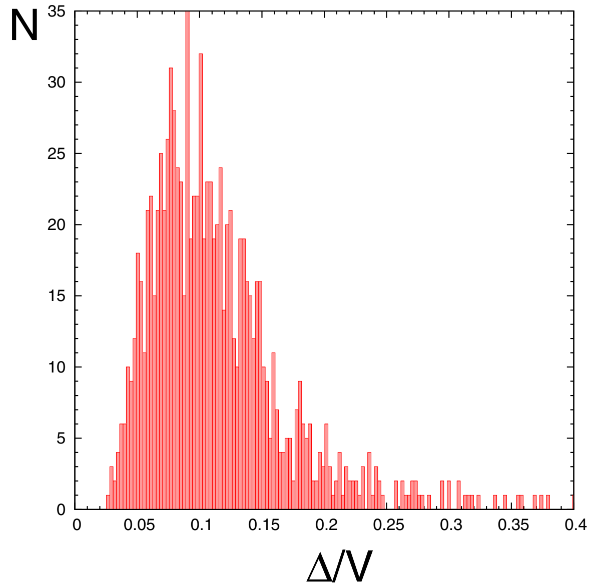

Fig. 8. (Color online) Distribution histogram of pair coupling energy $\Delta = 2E_F - E_g$ obtained from $N_D = 1000$ disorder realizations at $U = -2V$, $W = 5V$, $F = 0$, $L = 40$; here N gives a number of realizations found in a given cell of Δ/V, E_g is the ground state energy of a given realization obtained numerically from (3).

observed in experiments with a change of a magnetic field (see e.g. [17,18,20]). However, the distribution of values of pair coupling energy $\Delta = 2E_F - E_g$ is strongly peaked near its average value and in addition it has very sharp edge in Δ (see Fig. 8) so that a large fraction of localized pairs becomes delocalized approximately at the same static field that gives a sharp current growth for $V > V_c = F_c L$ (here E_g is a ground state energy for a given disorder realization at $F = 0$). Thus we can argue that in numerical simulations with many pairs all of them will be destroyed at approximately the same static field. This will produce an avalanche of delocalized electrons which will give a sharp jump in resistivity corresponding to experimental observations.

At that point we would like to note that even if our attraction is formally relatively strong (e.g. $|U| = 2V \sim V$) it effectively gives a rather weak coupling energy of BLS pairs $\Delta \approx 0.1V$ (see Fig. 8). There are a few physical reasons behind this. At first, a simple physical estimate gives a consistent value $\Delta \sim |U|/\xi \sim 0.05V \ll V$ where we use numerically found value of IPR $\xi \approx \ell^2 \approx 40$ from Figure 4. Thus the numerical value of Δ is by a factor 100 smaller than the energy band width of the noninteracting 2D problem $E_b \approx 8V + W \approx 14V$. Second, in (3) the summation over one-particle orbitals is done only inside a Debye energy interval $E_{ph} = \hbar\omega_D \approx 3.75V/\alpha \approx 0.25V \ll E_b$ and due to that the effective attraction is additionally decreased giving a relatively small coupling energy. Also in a good metallic phase (e.g. $W \leq 2V$) the interaction (e.g. even $|U| = 2V$) produces a quite weak effect on noninteracting delocalized states according to numerical results presented in [6,7] and according to usual theoretical estimates for interaction matrix elements $U_s \sim U/g \ll E_b$,

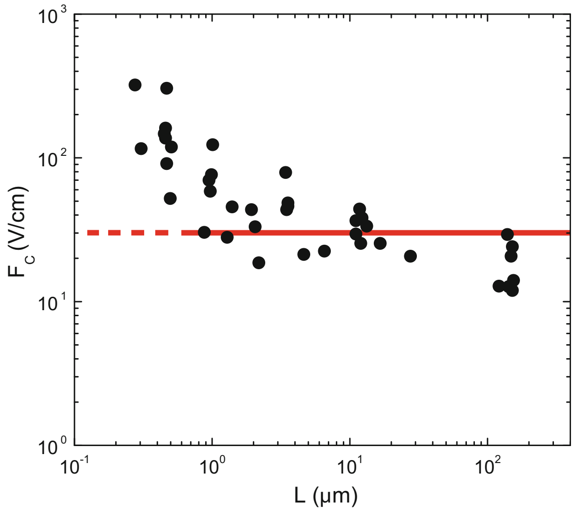

Fig. 9. (Color online) Critical field F_c for superinsulator destruction as a function of sample size L from experiments [19] (points from Fig. 5 there). The theory (1) is shown by red horizontal line.

which are inversely proportional to a sample conductance $g \gg 1$ (see e.g. [28,30] and references therein). Due to these reasons we think that the claim expressed in [3], that the BLS pairs appear only as a result of nonphysically strong attraction, is not justified since our BLS pairs have rather small coupling energy $\Delta \ll E_b$ and have rather large size $\xi \gg 1$. In addition, the numerical data presented in [6] (see also Fig. 2) show that the BLS pairs appear also at smaller values of attraction. Thus, even if, with the aim to have well localized pairs inside a system size available for numerical simulations, we fixed an attraction at a relatively strong value, our numerical data show that such a choice is still in the regime of weakly coupled pairs of relatively larger size that corresponds to the experimental regime. We note that even larger $|U|/V$ values are typically used in quantum Monte Carlo simulations (see e.g. [36]).

The main result of this studies is given by equation (1) which determines the critical voltage V_c of superinsulator destruction via the sample size L, BLS coupling energy Δ and pair size ℓ. According to (1) the critical voltage V_c is proportional to the sample size and hence, F_c is independent of L. This is in a good agreement with the experimental data obtained in [19] (see Fig. 9 with experimental points from Figure 5 in [19]). Indeed, in the range $0.5\ \mu m < L < 150\ \mu m$ the value of F_c shows significant fluctuations but in average remains constant. The average value is $F_c \approx 30V/\text{cm}$, and since the typical value of pair coupling energy is $\Delta \approx T_0 \approx 3{-}15$ K, we find the size of localized pairs to be $\ell = T_0/F_c = 100{-}500$ nm. The theory (1) is valid for $L > \ell$ where indeed F_c is independent of L, a part of fluctuations. However, for $L < \ell \sim 0.5\ \mu m$ one enters into another regime where the sample size becomes comparable with the pair size that can lead to an increase of F_c seen experimentally. It is clear that as soon as the superinsulator phase is destroyed at $F > F_c$ a further decrease of F below F_c places localized pairs in other new locations where due to fluctuations of disorder one gets a comparable but somewhat different new value of F_c. This leads to a hysteresis behavior observed experimentally.

For $F < F_c$ experiments [17,18,20] show very sharp temperature dependence which should be related to nontrivial hopping mechanisms of finite temperature conductivity of localized pairs with coupling energy Δ. For the experiments of Gantmakher et al. [16] at $F = 0$ the conductivity is well described by the Arrhenius law that can be compatible with the picture of BLS pairs. It is possible that correlations between BLS pairs should be taken into account to obtain more sharp temperature dependence. However, such type of analysis goes beyond our studies restricted to TIP approach.

It is also interesting to note that the relation (1) allows to determine the dependence of ℓ on magnetic field assuming $\Delta = \text{const}$. Indeed, in experiments [17,18,20] the values of L is known, and also $\Delta \approx T_0$ and V_c are experimentally known as a function of magnetic field B that allows to determine the dependence $\ell(B)$ from the relation (1). However, a sufficiently strong magnetic field B also affects a strength of effective attraction since it breaks time-reversibility and gives a decrease of U destroying superconductivity at weak B. Also B gives and increase of returns to a same impurity and effective increase of disorder. A similar arguments are given in Section 2 (see dashed curve in Figure 2 and physical arguments given for it there). Thus at a fixed static field F one enters from superconductor to superinsulator by increasing B from zero to finite moderate values, that corresponds to the experimental observations (see e.g. Fig. 3a in [20]). However, at even larger values of B an effective attraction is completely eliminated and one enters in the noninteracting metallic phase with disappearance of superinsulator in agreement with experimental observations (see e.g. Fig. 3a in [20] at larger B). This physical picture globally corresponds to those described in Section 2 for the explanation of Gantmakher et al. experiments with related Figure 2.

On the basis of presented results we can draw a global phase diagram in the temperature-disorder plane shown in Figure 10 for a fixed attraction strength (e.g. at $U = -2V$), zero static field and fixed Fermi energy E_F. At small disorder and temperature we have the superconducting phase S which is followed by a transition to metallic phase M at large temperature or to the localized BLS phase at low temperature. At a larger disorder $W > W_c$, but still small temperatures, the BLS phase enters in the insulating regime of noninteracting Anderson insulator I. In this regime with $W > W_c$ but temperature above a certain threshold $T > T_2 \sim |U|(1/\xi + 1/\xi_1)$, the TIP pairs become delocalized by interactions with emergence of metallic phase of TIP delocalized pairs, as it is discussed in [27,28,30]. Here ξ_1 is a noninteracting one-particle IPR which gives a dominant contribution for a disorder $W > W_c$ which is not very close to the critical point $W = W_c$, the term $1/\xi$ with IPR of BLS is included to have interpolation between two phases.

In conclusion, we presented the BLS based physical picture for a destruction of superinsulator by a finite static field observed experimentally in [17–21]. This picture is

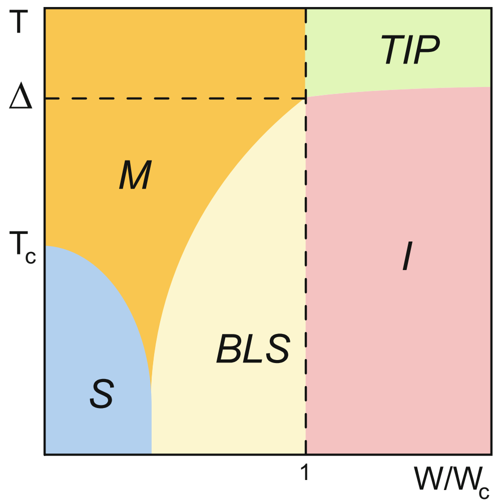

Fig. 10. (Color online) Schematic phase diagram in the temperature-disorder plane (T, W) for quasi-2D or 3D Anderson model at fixed Hubbard attraction $U = -2V$ and fixed filling factor with the Fermi energy E_F: the vertical dashed line shows the Anderson transition at $W = W_c \approx 16.5V$ for noninteracting particles, S is the superconducting phase, BLS is the insulator phase of localized BLS pairs, M is essentially the noninteracting metallic phase, TIP is the metallic phase of delocalized TIP pairs. Here we assume $T \ll E_F$, static field is zero, temperature linear axis is shown in arbitrary units.

rather different from other theoretical explanations discussed in [20,22–26]. The main new element of our theory is the existence of localized pairs created by attraction inside noninteracting metallic phase which is absent in the above theoretical proposals. In our theory the BLS phase is the basis of superinsulator and since the noninterating states are metallic this phase is sharply broken by a finite static field which breaks electron pairs, localized by attraction, and lets them propagate like noninteracting particles in a metal. Our analysis considers a breaking of only two interacting particles. In a real system with many pairs a breaking of a few pairs can create a strong avalanche and breaking of other pairs so that a critical static field can be determined by breaking of mostly weakly coupled pairs with a smaller critical fields, compared to average field values found here. This physical picture is rather different from the superinsulator picture discussed in [20]. However, we keep the term superinsulator which in our opinion nicely describes impressive experiments [17,18,20,21] with superconducting films, which become insulating in magnetic fields with abrupt emergence of current above a certain critical *dc*-voltage.

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