Chaotic dark matter in the Solar system and galaxies

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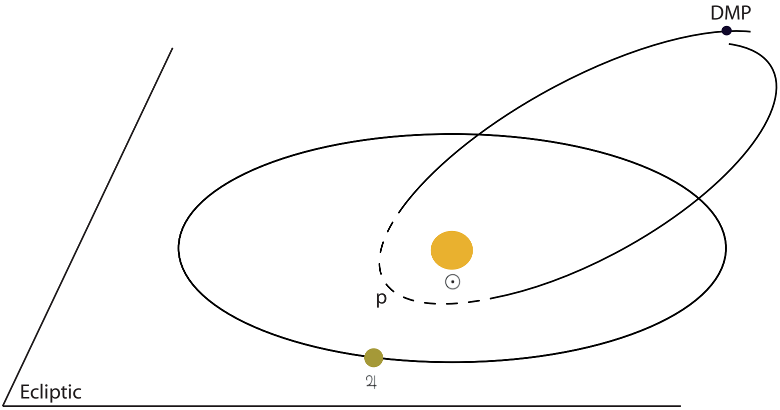
References :

J. L., D. Shepelyansky, **Dark matter chaos in the Solar system**, MNRASL 430, L25-L29 (2013), arXiv :1211.0903
G. Rollin, J. L., D. Shepelyansky, **Chaotic enhancement of dark matter density in binary systems and galaxies**, to be submitted

Dark matter capture – Three-body problem



Possible DMP capture due to Jupiter rotation around the Sun

Dark matter capture – Restricted circular three-body problem

$$m_{\text{DMP}} \ll m_{♃} \ll m_{\odot}$$

Dark matter capture – Restricted circular three-body problem

Newton's equations

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$$\ddot{\mathbf{r}} = \frac{1 - m_{♃}}{\left\|\mathbf{r}_{\odot}(t) - \mathbf{r}\right\|^3}\left(\mathbf{r}_{\odot}(t) - \mathbf{r}\right) + \frac{m_{♃}}{\left\|\mathbf{r}_{♃}(t) - \mathbf{r}\right\|^3}\left(\mathbf{r}_{♃}(t) - \mathbf{r}\right)$$

$$G = 1, \quad m_{♃} + m_{\odot} = 1, \quad \left\|\dot{\mathbf{r}}_{♃}\right\| \simeq 13\text{km.s}^{-1} = 1, \quad \left\|\mathbf{r}_{♃}\right\| = 1$$

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Energy change after a passage at perihelion (in absence of close encounter)

$$F \sim \frac{m_{♃}}{m_{\odot}} \left\|\dot{\mathbf{r}}_{♃}\right\|^2 \simeq 10^{-3} \qquad \text{(Petrosky 86')}$$

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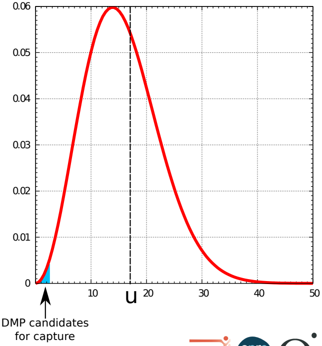
Assuming a Maxwellian distribution of Galactic DMP velocities

$$f(v)dv \sim v^2 \exp\left(-3v^2/2u^2\right) dv$$

with $u \simeq 220\text{km.s}^{-1} \sim 17$ (mean DMP velocity)



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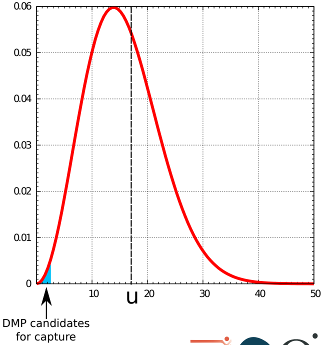
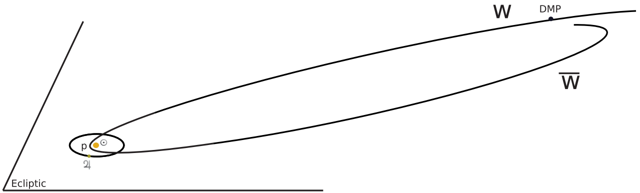


As $F \ll u^2$, not many candidates for capture among Galactic DMPs

Most of the capturable DMPs have close to parabolic approaching trajectories ($E \sim 0$)

Direct simulation of Newton's equations is difficult : very elongated ellipses, not many particles can be simulated, CPU time consuming (Peter 09')

Dark matter capture – Kicked model



x : Jupiter's phase when DMP at perihelion ($x = \varphi/2\pi \mod 1$)
w : DMP energy ($w = -2E/m_{\text{DMP}}$)

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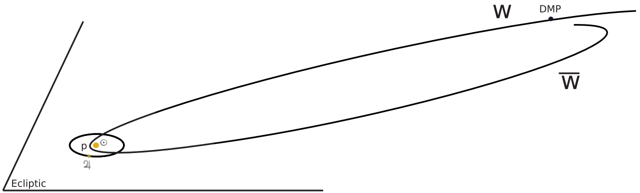

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Symplectic Kepler-Petrosky map

$$\bar{x} = x + \bar{w}^{-3/2} \quad \text{third Kepler's law}$$
$$\bar{w} = w + F(x) \quad \text{energy change after a kick}$$

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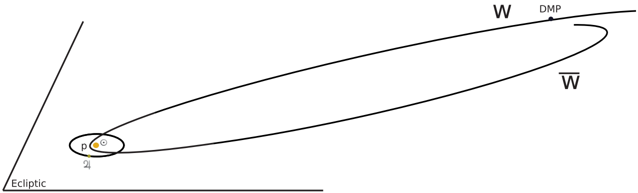

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Map already used in the study of :

- Cometary clouds in Solar systems (Petrosky 86')
- Chaotic dynamics of Halley's comet (Chirikov, Vecheslavov 89')
- Microwave ionization of hydrogen atoms (see e.g. Shepelyansky, scholarpedia)

Advantage : providing the fact the kick function $F(x)$ is known the dynamics of a huge number of particles can simulated.

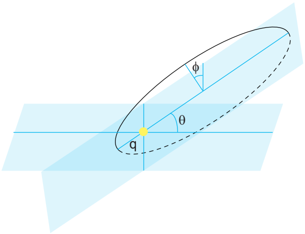
Dark matter capture – Dark map

Kick function determination

$$F(x) \to F_{\ell,\theta,\phi}(x) \sim F_{q,\theta,\phi}(x)$$

The kick function is different for each approaching configuration $(\ell, \theta, \phi) \sim (q, \theta, \phi)$
For close to 1 eccentricities the perihelion is such as

$$q \sim \frac{\ell^2}{2}$$



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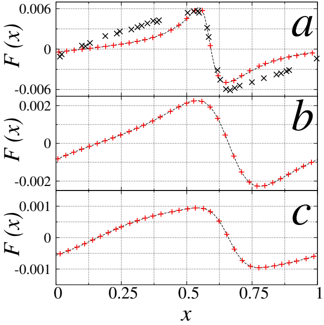
For each approaching configuration (q, θ, ϕ), the integration of the three-body problem gives the kick function.



a : Halley's comet
b : $q = 1.5, n = 4, \theta = 0.7, \phi = 0$
c : $q = 0.5, n = 4, \theta = 0., \phi = \pi/2$

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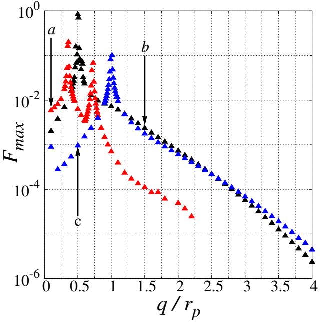
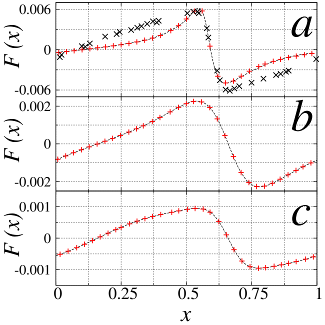


Exponential decay in agreement with Petrosky 86'



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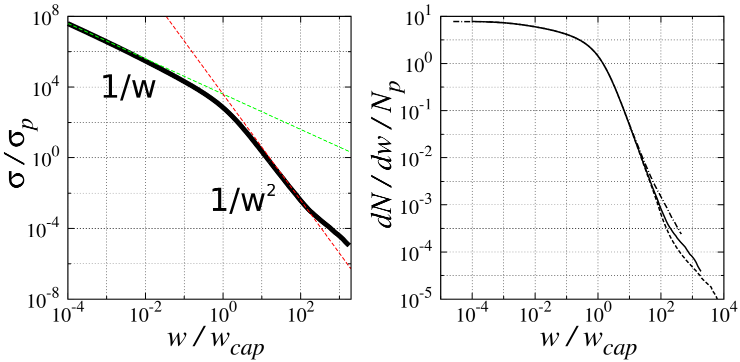
Dark matter – Capture cross section

$w_{cap} = \frac{m_{♃}}{m_{\odot}} \left\|\dot{\mathbf{r}}_{♃}\right\|^2 \simeq 10^{-3}$ $\sigma_p = \pi \left\|\mathbf{r}_{♃}\right\|^2$ Jupiter orbit area

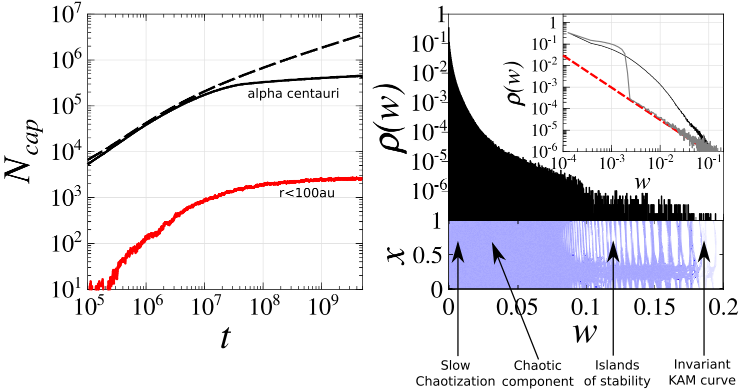


$\sigma / \sigma_p \simeq \pi \frac{m_{\odot}}{m_{♃}} \frac{w_{cap}}{w}$ in agreement with Khriplovich & Shepelyansky 09'

- Predominance of long range interaction as suggested by Peter 09'
- Very small contribution from close encounters invalidating previous numerical results (Gould & Alam 01' and Lundberg & Edsjö 04')

Dark matter evolution – Chaotic dynamics

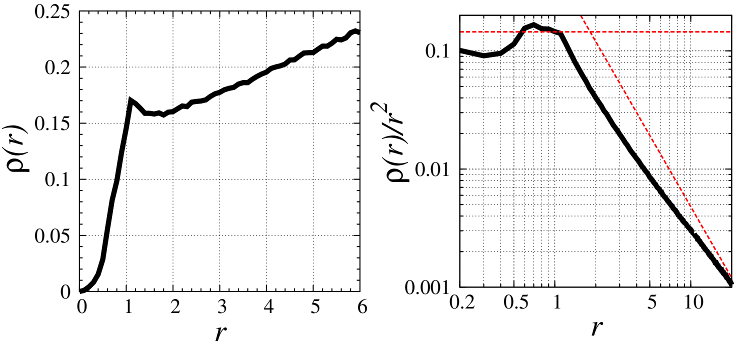
Simulation of the (isotropic) injection, the capture and the escape of DMPs during the whole lifetime of the Solar system.
Injection of $N_{tot} \simeq 1.5 \times 10^{14}$ DMPs with energy $|w|$ in the range $[0, \infty]$ with $N_H = 4 \times 10^9$ DMPs in the Halley's comet energy interval $[0, w_H]$.



- Equilibrium reached after a time $t_d \sim 10^7$yr similar to the diffusive escape time scale of the Halley's comet (Chirikov & Vecheslavov 89') $\longrightarrow$ Equilibrium energy distribution $\rho(w)$
- The dynamics of dark matter particles in the Solar system is essentially chaotic

Back to real space – Density distribution of captured DMPs

Nowadays equilibrium density distribution ($t_S = 4.5 \times 10^9$yr)



- **The profile of the radial density $\rho(r) \propto dN/dr$ is similar to those observed for galaxies where DMP mass is dominant.** Indeed $\rho(r)$ is almost flat (increases slowly) right after Jupiter orbit $(r = 1) \longrightarrow$ according to virial theorem the circular velocity of visible matter is consequently constant as observed e.g. in Rubin 80'
  More precisely, $v_m \propto r^{0.25}$ (Dark map) quite close to $v_m \propto r^{0.35}$ (Rubin 80')

Back to real space – Density distribution of captured DMPs

Surface density

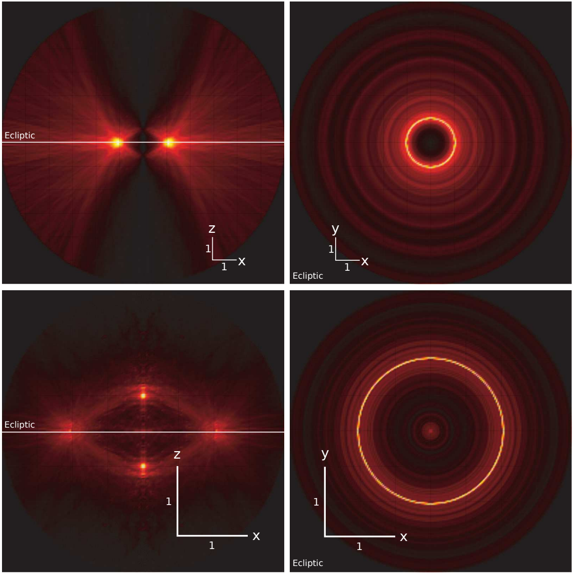
$$\rho_s(z, R) \propto dN/dzdR$$

where

$$R = \sqrt{x^2 + y^2}$$

Volume density

$$\rho_v(x, y, z) \propto dN/dxdydz$$



How much dark matter is present in the Solar system ?

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The total mass of DMP passed through the System solar during its lifetime $t_S = 4.5 \times 10^9$yr is

$$M_{\text{tot}} = \rho_g t_S \int_0^\infty dv\, v f(v)\sigma(v) \approx 35\rho_g t_S G \left\|\mathbf{r}_{♃}\right\| M_\odot / u \approx 0.9 \times 10^{-6} M_\odot \sim M_{♀}$$

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At time t_S the mass of captured DMPs in the Solar system is

$$M_{AC} \approx \eta_{AC} M_{tot} \approx 2 \times 10^{-15} M_\odot \quad \text{within } r < 0.5 \text{ distance}_{\text{Sun-}\alpha\text{Centauri}}$$
$$M_{100\text{au}} \approx \eta_{100\text{au}} M_{tot} \approx 1.3 \times 10^{-17} M_\odot \quad \text{within } r < 100\text{au}$$

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$$\rho_{gH} = \rho_g \int_0^{\sqrt{w_H}} dv\, v f(v) \approx 1.4 \times 10^{-32}\text{g/cm}^3$$

Huge chaotic enhancement $\zeta = \rho_{♃}/\rho_{gH} \approx 4 \times 10^3$ of the density of actually capturable DMPs.

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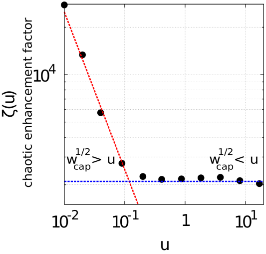
$\Longrightarrow$

The long range interaction capture mechanism is very efficient for binary systems (1+2) with $m_1 \gg m_2$

Huge chaotic enhancement $\zeta = \rho_{♃}/\rho_{gH} \approx 4 \times 10^3$ of the density of actually capturable DMPs.

Conclusion : Dark matter capture in binary systems (preliminary results)

$w_{cap} \sim w_H \sim 0.005$
$u = 220 km/s$



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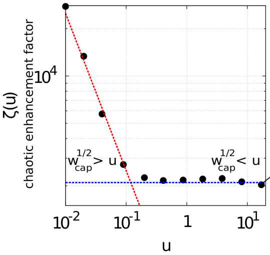

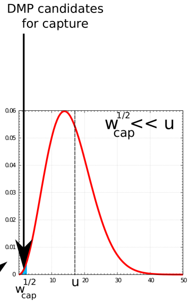

e.g.
Sun
+
Jupiter

$V_{Jupiter} \sim 13\text{km/s}$

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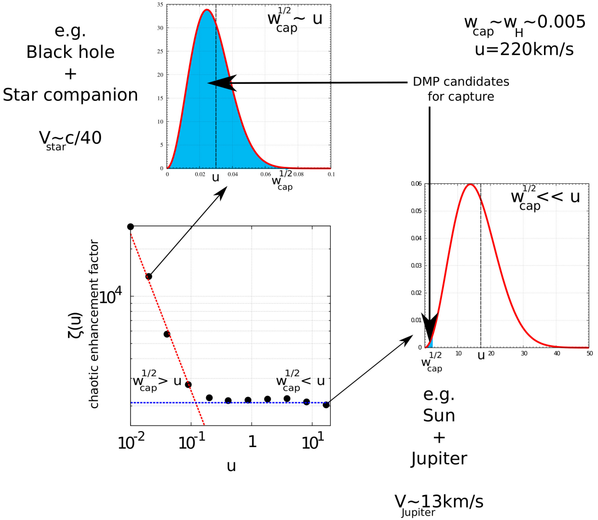

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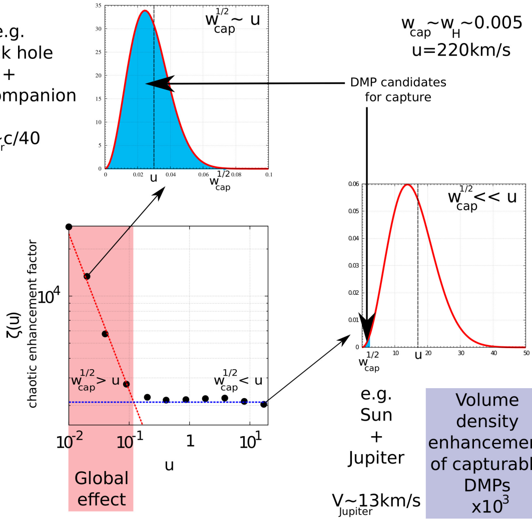
Global
volume density
enhancement
x10^4

all the galactic DMPs are captured

e.g.
Black hole
+
Star companion

$V_{star} \sim c/40$



e.g.
Sun
+
Jupiter

$V_{Jupiter} \sim 13$km/s

Volume
density
enhancement
of capturable
DMPs
x10^3

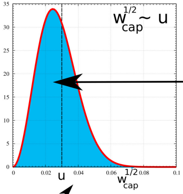
Conclusion : Dark matter capture in binary systems (preliminary results)

Global
volume density
enhancement
$\times 10^4$

all the galactic DMPs are captured

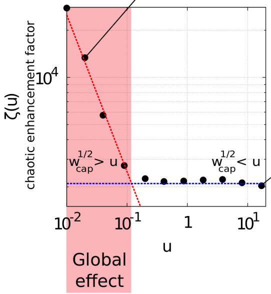
e.g.
Black hole
+
Star companion

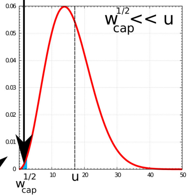
$V_{star} \sim c/40$



$w_{cap} \sim w_H \sim 0.005$
u=220km/s

DMP candidates
for capture





e.g.
Sun
+
Jupiter

$V_{Jupiter} \sim 13$km/s

Volume
density
enhancement
of capturable
DMPs
$\times 10^3$

Ionization process:
- DMPs acceleration
- wandering black holes
- ...

Thank You !