Chaos and Philosophy

Guillaume Rollin

APEX (kick-off meeting)

20/10/2017

Outline

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The Chirikov Standard Map

- Constructed by a Poincaré's surface of section of the kicked rotator.

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\begin{aligned}
I_{n+1} &= I_n + K\sin(\theta_n)(mod\ 2\pi) \\
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\end{aligned}
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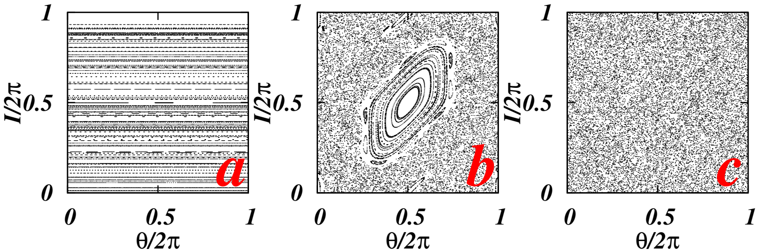
- It gives a Poincaré's section of the form :



Chirikov standard map for a) $K = \mathbf{0}$, b) $K = \mathbf{1.928}$, c) $K = \mathbf{7}$.

- With the increase of the perturbation, the chaos destroys the stable islands.

References :
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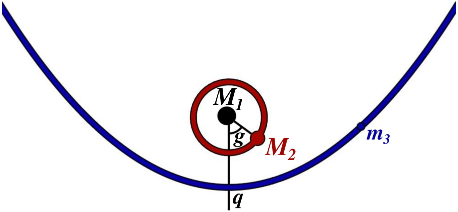
The Kepler Map (1/2)



g is the planet phase, q is the perihelion position, M_1 is the primary mass M_2 is the secondary mass

Assumptions :

- $M_1 \gg M_2 \gg m_3$: for example, comet travelling through the Solar System constituted by the Sun and Jupiter
- quasi-parabolic motion ($E \sim 0$, $e \sim 1$)
- large perihelion distance $q \gg 1$ (to avoid close encounters)

Solution : The Kepler Map

$$\begin{aligned} P_{n+1} &= P_n + A\sin(g_n) \\ g_{n+1} &= g_n - 2\pi\sigma/(-P_{n+1})^{3/2} \end{aligned}$$

References :
T. Y. Petrosky, Phys. Letters A, 117(328), 1986.
T. Y. Petrosky and R. Broucke, Celestial Mechanics, 42 :53–79, 1988.

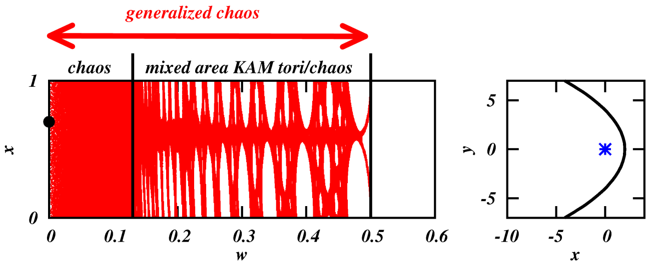
The Kepler Map (2/2)

- We use Chirikov and Vesheslavov ('89) form of the Kepler map.

$$w_{n+1} = w_n + F(x_n)$$
$$x_{n+1} = x_n + w_{n+1}^{-3/2}$$

where $w = -2E/m_c$

- Poincaré Section in phase space :



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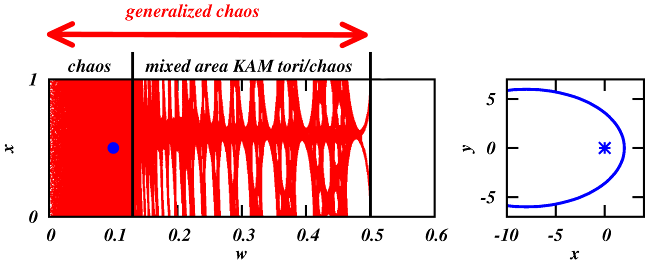

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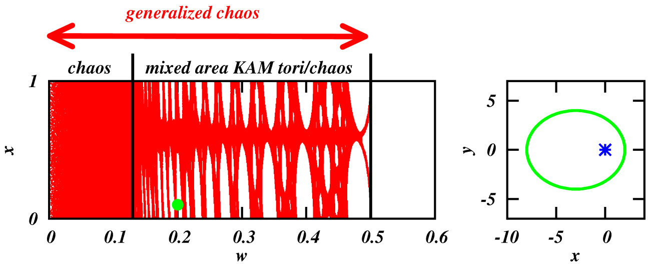

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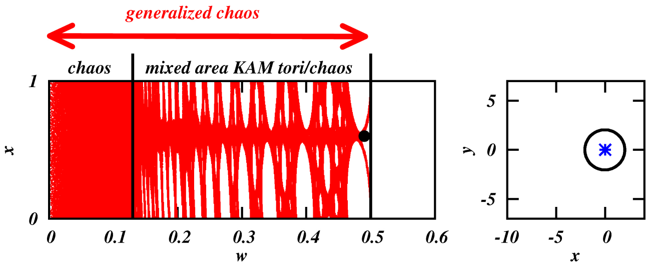

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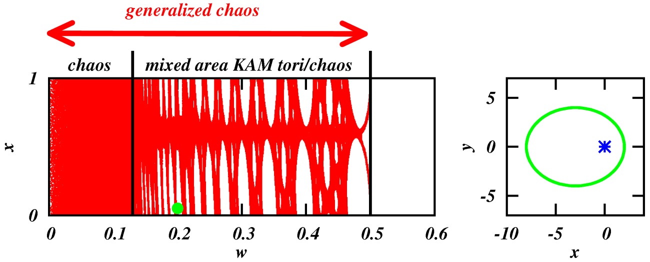

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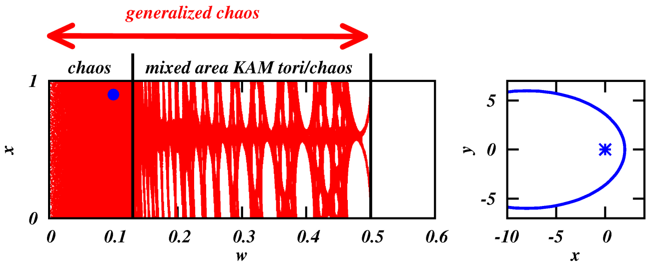

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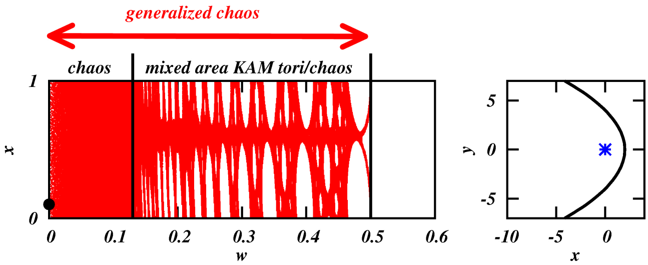

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The Chaos Limit (1/5)

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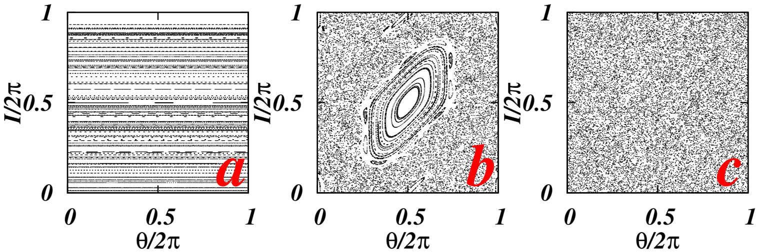

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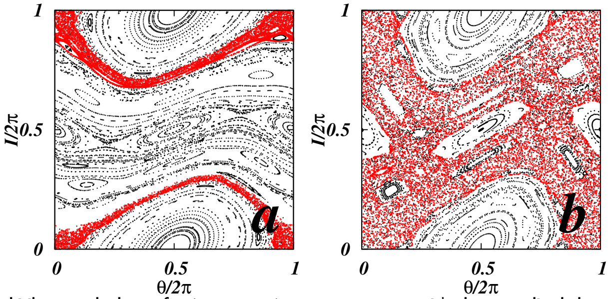
- The Chirikov Criterion is given by :

$$K_{ch} \sim 1 \quad (1)$$

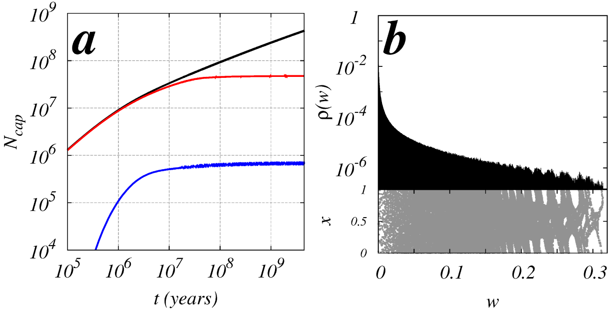


Chirikov standard map for a) $K = 0.8$, b) $K = 1.2$. For $K \sim 1$ it's the generalized chaos.

References :
B. V. Chirikov, Phys. Rep., 52 :263, 1979

The Chaos Limit (3/5)

- Capture and associated dynamics in the circular restricted 3-body problem.



a) **Number of captured particles with time.** b) **Energy distribution during the steady state.**

- We can express the chaos border by using the Chirikov criterion :

$$w_{ch} = (3\pi J)^{2/5} \simeq 0.29$$

- Moreover, we can represent this limit on a $e - q$ stability diagram.

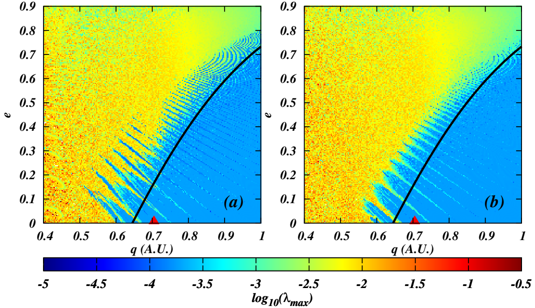
References :
G. Rollin, J. Lages, and D. L. Shepelyansky, A&A, 576 :A 40, 2015

The Chaos Limit (4/5)

- $e - q$ stability diagram : we use Lyapunov exponent method.
- The formulation of the chaos border found by Shevchenko is :

$$e_{ch} = 1 - 2q\Delta E_{ch}$$
$$\Delta E_{ch} \simeq A\mu^{2/5} q^{-1/10} \exp\left(-Bq^{3/2}\right)$$

with $A = 2^{-1/2} 3^{2/5} \pi^{3/5} K_G^{-2/5}$ and $B = 2^{5/2}/15$, here $K_G = 0.971635406\ldots$.



One exemple of e-q stability diagram for Kepler 16b.

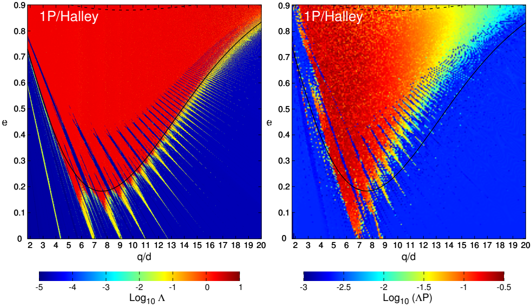
References :
I. I. Shevchenko, ApJ, 799:8 (7pp), 2015
E. A. Popova and I. I. Shevchenko, ApJ, 769:152 (7pp), 2013

The Chaos Limit (5/5)

- We can study the same problem for a rotating system like comet nuclei.

$$
\begin{aligned}
E_{i+1} &= E_i + \Delta E(\phi_i) \\
\phi_{i+1} &= \phi_i + \frac{2\pi\omega}{\mid 2E_{i+1} \mid^{3/2}}
\end{aligned}
$$

- If we conserve only the two first leading terms. The disruptive function is given by $\Delta E(\mu, q, \omega, \phi) \simeq W_1 \sin(\phi) + W_2 \sin(2\phi)$. Where W_1 and W_2 decrease exponentially with q, the pericentric distance.



Dynamics around Halley's comet nuclei.

References :
J. Lages, D. L. Shepelyansky and I. I. Shevchenko, AJ, 2017
J. Lages, I. I. Shevchenko and G. Rollin, *in press*

Outline

The Free Will Problem

		ARE WE FREE TO ACT ?	
		YES	NO
IS DETERMINISM COMPATIBLE WITH FREE WILL ?	YES	COMPATIBILISM	?
	NO	LIBERTARIANISM	PESSIMISM, HARD DETERMINISM

Main visions about the Free Will problem.

- **Determinism :** Any event is necessitated by the events of the past in conjunction with the laws of nature.
- **Compatibilist :** The Free Will is compatible with the determinism
- **Libertarian :** The Free Will is incompatible with the determinism and the determinism is incorrect. Libertarians needs indeterminisism in the causal chain $\rightarrow$ QM ?
- **Hard determinist :** The Free Will is incompatible with the determinism but the derterminism is correct $\rightarrow$ Free Will is impossible.

References :
E. Pasherie, Institut Jean Nicod, CNRS-ENS-EHESS, Paris
http://pacherie.free.fr/COURS/FCS1-2011-cours2.pdf

The importance of the concept of Chaos

- Libertarians need indeterminism : classical QM interpretation seems to give it.
- **Problem** : in brain environment, a quantum state seems to be destroyed very quickly.
- To export quantum indetermination, the Libertarians suggest a conjunction between QM and Chaos.
- Idea :
  1. Keep the concept of chaos for the cerebral system.
  2. Reject the Free Will concept based on an indeterminism vision.
- Chaos seems to give a good interpretation of the Free Will in the hard deterministic vision... → W.I.P

References :
P. Faure and H. Korn, Is there chaos in the brain ? I. Concepts of nonlinear dynamics and methods of investigation, C.R. Acad. Sci. Paris, Sciences de la vie / Life Sciences 324 (2001) 773–793
H. Korn and P. Faure, Is there chaos in the brain ? II. Experimental evidence and related models, C. R. Biologies 326 (2003) 787–840