A Complex Network Approach of Go Game

APEX PROJECT KICK-OFF MEETING
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I The Game of Go

- Game of go: very ancient Asian game, probably originated in China in Antiquity
- Different name for different country :

  Japan = Go

  China = Weiqi

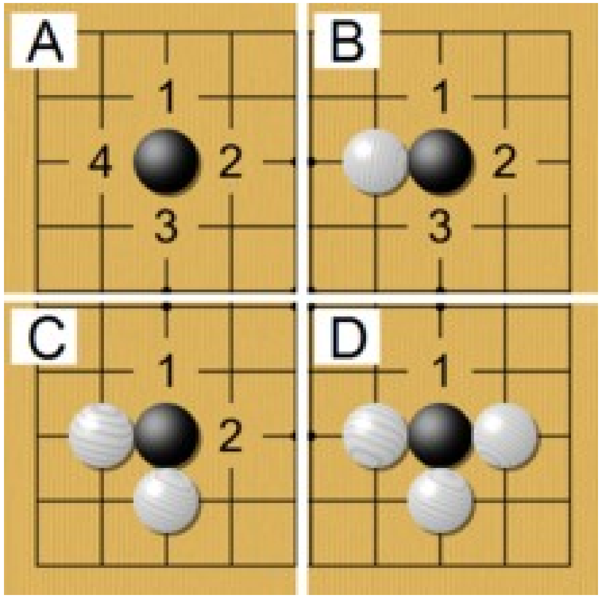
  Korea = Baduk

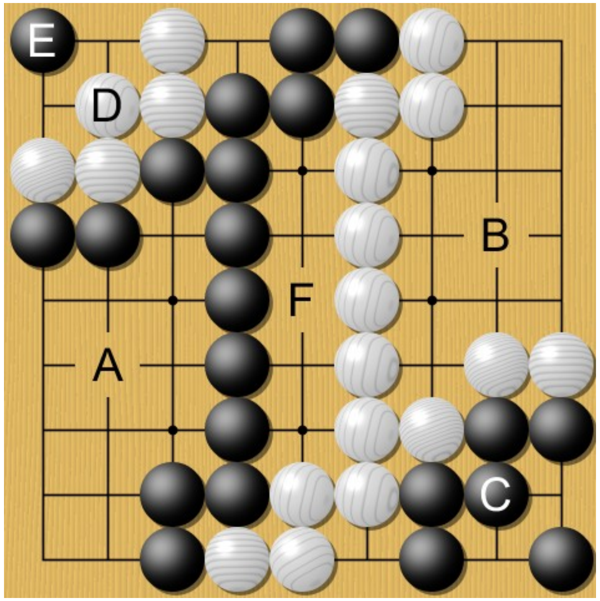
I The Game of Go

- Go is a very popular game in asia, this game is payed on a goban (see below)

1.1 Rules of Go

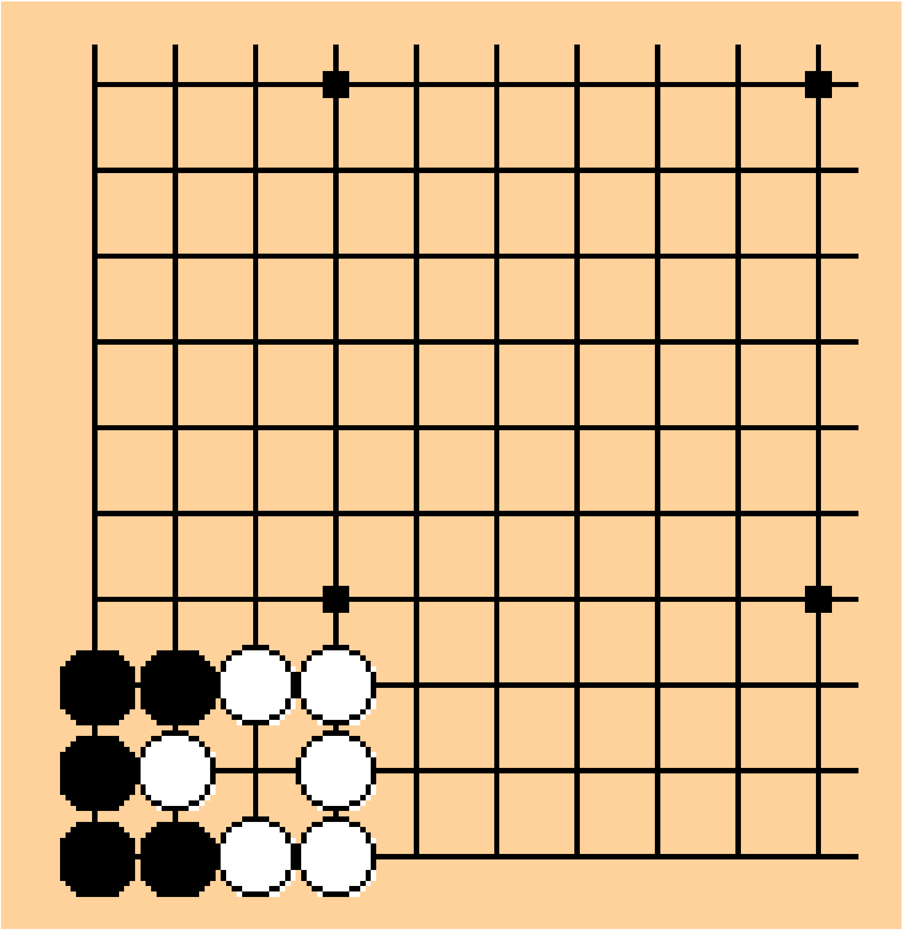
- White and black stones alternatively put at intersections of *19 X 19 lines*
- Stones without liberties are removed
- A chain with only one liberty is said in atari
- Handicap stones can be placed
- Aim of the game : construct protected territories

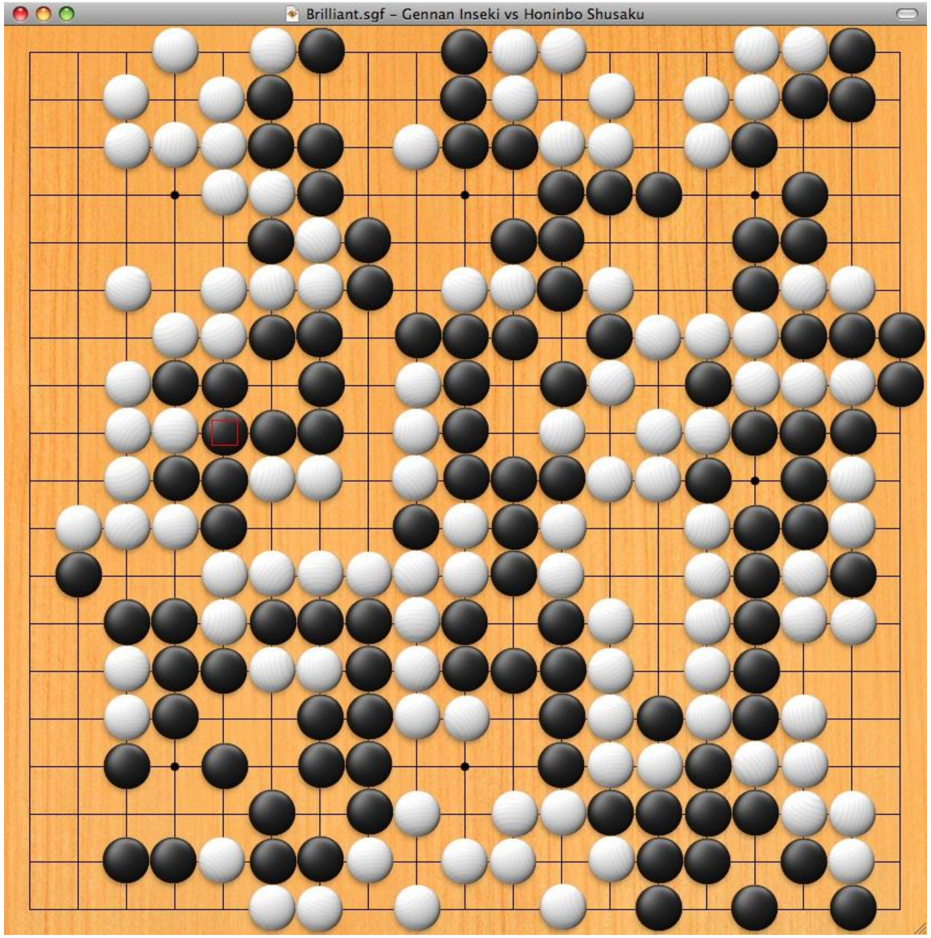




1.1 Rules of Go

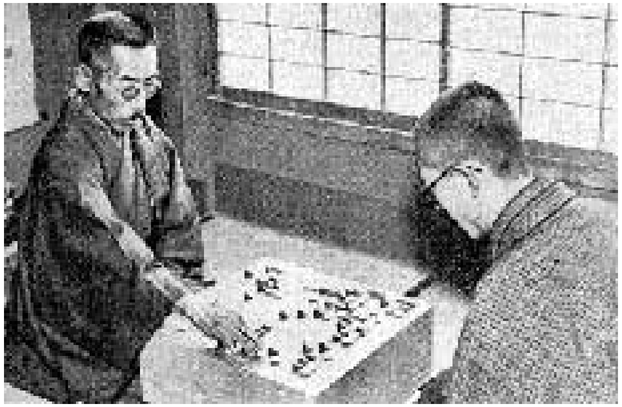
- A ko (left) and endgame (right) exampe :





1.2 Player rankings

- There are nine levels (dans) of professionals followed by nine levels of amateurs
- A handicap stone can compensate for roughly one dan: like in golfing, players of different levels can play evenly thanks to handicaps
- There are regular tournaments of go since very long times



1.3 Computer Simulations

- While Deep Blue beat the world chess chamion Kasparov in 1997, Only since 2016 a computer program (AlphaGo) has beaten one of the best go player:

  March 2016: It wins *4-1* vs. Lee Sedol (world No.3 ranked player)

  May 2017: It wins *3-0* vs. Ke Jie (world No.1 ranked player)

- Difficult game to simulate:

  Total number of legal positions 10^{171} vs. 10^{50} for chess

  Not easy to assign positional advantage to a move

- AlphaGo uses Monte Carlo tree search algorithm and deep learning techinques, It can play random games during a game in order to assigned a value to a move

1.4 Databases

- Human played games :

  8000 amateur games (*http://www.u-go.net/*)

- Computer generated games :

  8000 games with deterministic algorithm (Gnugo)

  8000 games with Monte Carle search tree algorithm (Fuego)

  Only 50 AlphaGo vs AlphaGo Games (*http://senseis.xmp.net/?AlphaGo*)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(;GM[1]
FF[4]
SZ[19]
PW[sususu]
WR[8d]
PB[coolbabe]
BR[5d]
DT[2009-12-15]
PC[The KGS Go Server at http://www.gokgs.com/]
KM[0.50]
RE[B+Time]
RU[Japanese]
OT[5x10 byo-yomi]
CA[UTF-8]
ST[2]
AP[CGoban:3]
TM[0]
HA[4]
AB[dd]
[pd]

[dp]
[pp]
;W[qj];B[qm];W[hr];B[qh];W[kc];B[oj];W[ch];B[fc];W[cn];B[ck]
;W[mp];B[bp];W[fn];B[bn];W[bm];B[cm];W[dm];B[cl];W[nc];B[dn]
;W[pi];B[nd];W[od];B[oc];W[oe];B[mc];W[nb];B[pc];W[ne];B[md]
;W[mb];B[pe];W[me];B[of];W[oi];B[ni];W[nj];B[ok];W[nk];B[ol]
;W[pg];B[ph];W[oh];B[qg];W[nl];B[nm];W[mm];B[mn];W[lm];B[ln]
;W[kn];B[nn];W[ko];B[km];W[lk];B[iq];W[hq];B[io];W[jp];B[ip]
;W[jm];B[kl];W[kk];B[jl];W[im];B[ll];W[ml];B[mj];W[mk];B[jk]
;W[nh];B[go];W[fo];B[gp];W[eq];B[gq];W[gr];B[hm];W[il];B[jj]
;W[mi];B[gl];W[hk];B[ep];W[gk];B[fl];W[dq];B[cq];W[ir];B[jq]
;W[cr];B[br];W[dr];B[jr];W[fq];B[fp];W[gd];B[gf];W[fd];B[ef]
;W[de];B[ed];W[ee];B[fe];W[cd];B[gc];W[hd];B[cc];W[ce];B[hc]
;W[id];B[bc];W[fg];B[fh];W[ei];B[eh];W[dh];B[gg];W[fi];B[gi]
;W[eg];B[gh];W[fj];B[ii];W[ig];B[di];W[dj];B[dg];W[cg];B[ff]
;W[df];B[eg];W[ih];B[ik];W[hl];B[gj];W[fk];B[hj];W[hh];B[jh]
;W[ki];B[ji];W[hi];B[jf];W[jg];B[kg];W[if];B[kh];W[kf];B[lf]
;W[ke];B[lh];W[mg];B[mf];W[le];B[li];W[kj];B[nf];W[lg];B[mh]
;W[ng];B[ni];W[pf];B[qf];W[mi];B[bf])
```

II Complex Directed Network Model

- $G := (E, V)$ is a network (graph) composed of N_E links (Edges) linking N_V nodes (Vertices)
- Each link has a direction from the outgoing node i toward the incoming node j.
- A first statistical investigation is the integrated distribution of links in a network :

  The degree $K(i)_{in/out}$ is the number of link (incoming or outgoing) of the i-th node

  $P(K_{in/out})$ is the probability to having at least K links (in/out) for a given node

  => Classification of networks, ex : (Scale-free, Random..)

II Complex Directed Network Model

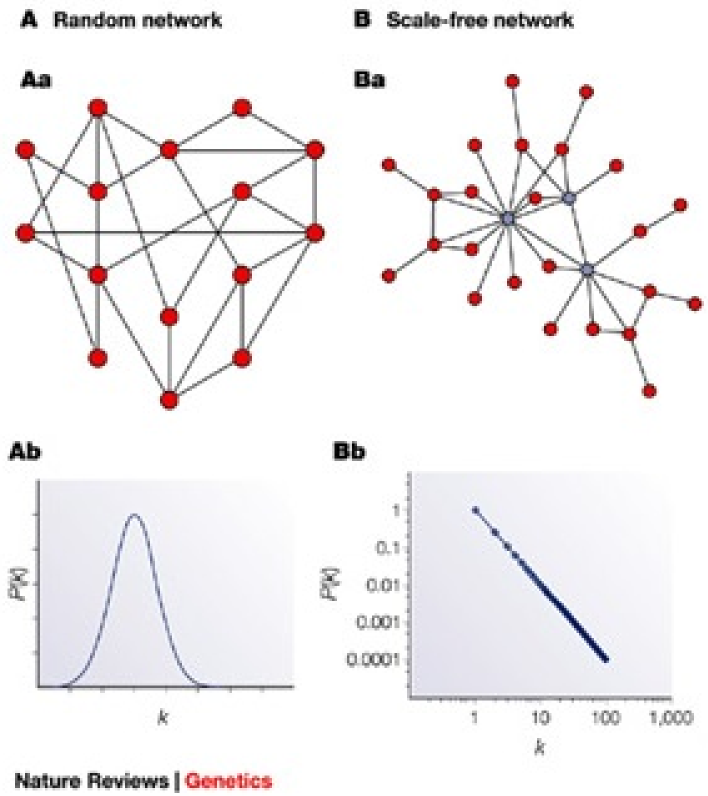


Fig. 1: Two types of networks (Top), their respectiv integrated links distribution (Bottom)

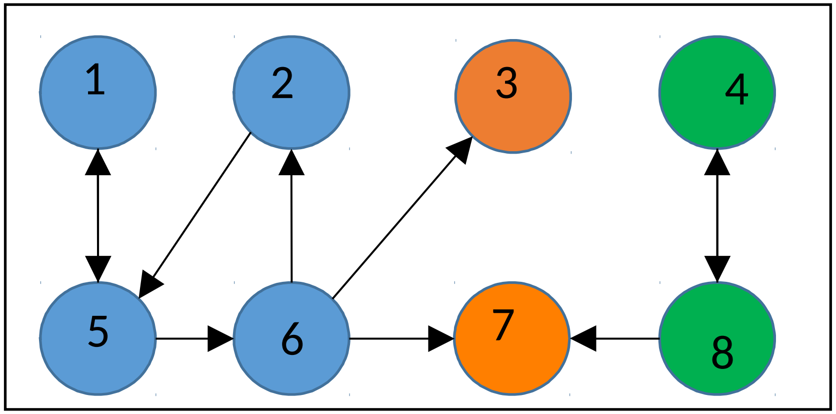


Fig. 2: Directed network, 8 nodes and 10 links. dangling node and dangling group

$P(k_{in/out}) = k^{\gamma}$ => Power law => Scale-free network

Particularities :

Hubs (Nodes with highest degree)

Small-world phenomenon

Examples : Social networks, protein-protein interactiton, WWW and semantic network

2.1 Nodes and local fight pattern

- We propose to use square of 3X3 intersections

N_v=*1107* nonequivalent patterns with empty centers:

symmetrically different

different by color swapping

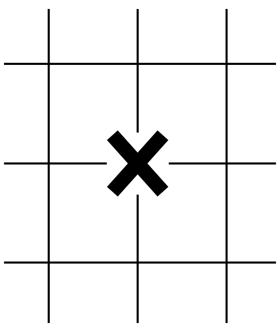


Fig. 3: Node ''0''

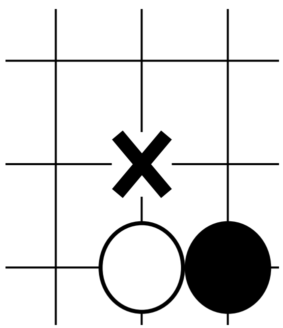


Fig. 4: Node ''7''

2.2 Links and strategy bias

- We propose to use $d_s = 4$ as a strategic distance between two linked nodes
- Time-directed links
- Finnaly if the node j (h_j, v_j) is played after (during the same game) the node i (h_i, v_i) and if we have $\boxed{\max\{\, |h_j - h_i|, |v_j - v_i| \,\} \leq d_s}$

2.3 A Scale-free Network

- Integrated distribution of links for each kind of network (Human, Gnugo and Fuego) represents scale-free networks with high symmetry between in/out links ($\gamma = -1$)
- Due to the construction method => as we take consecutive games within a database for our network a node is often a source and a destination of a link
- Value of K for hubs (rightmost points) within human network is higher than in both computers network => human seems to prefer certain moves independently of the global strategy
- There are more oscillations for Gnugo and Fuego

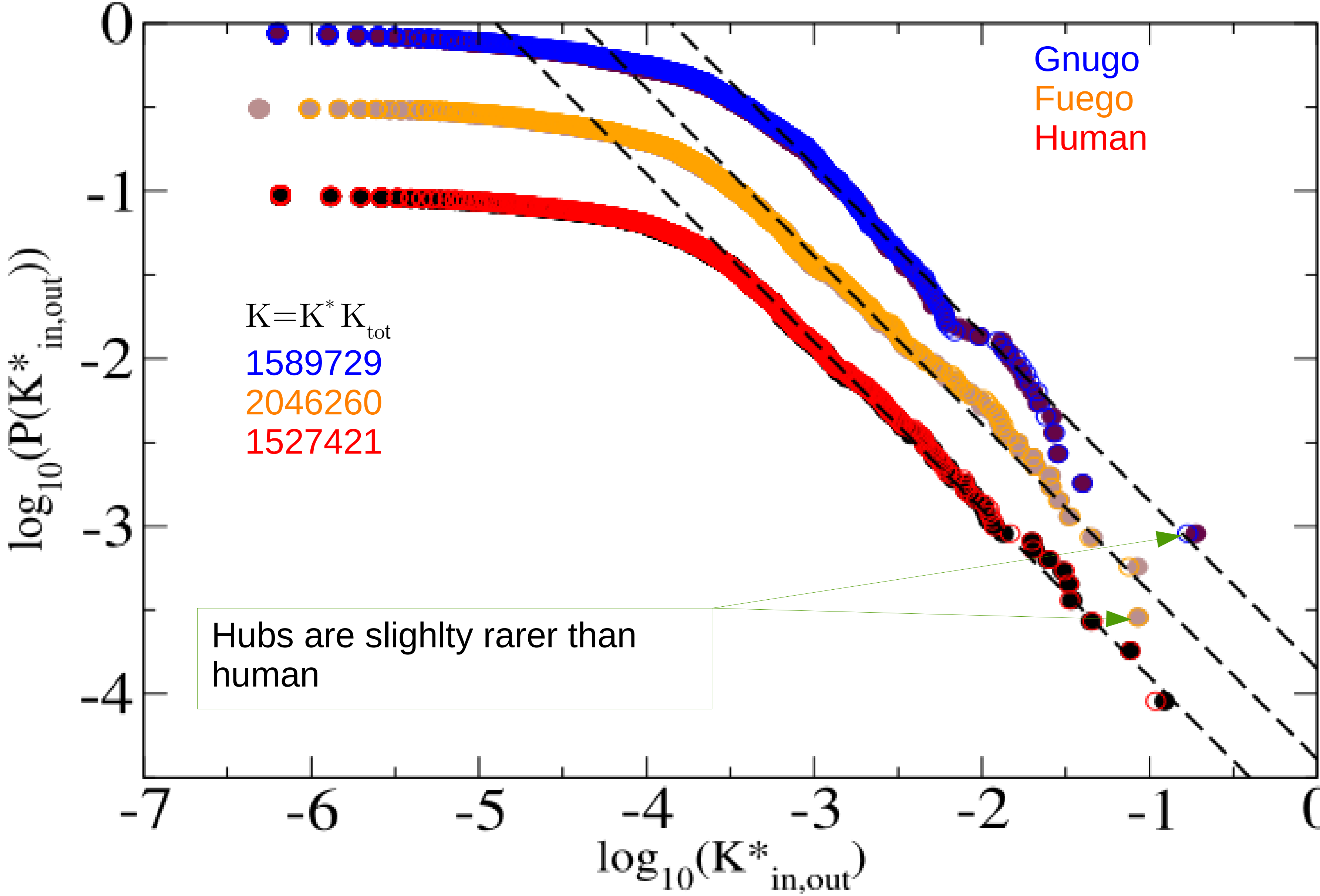


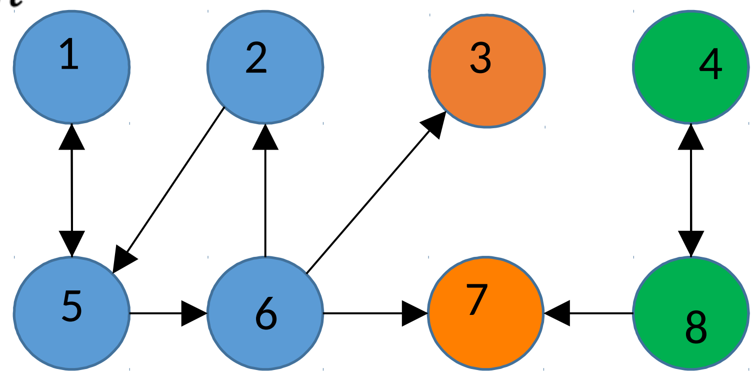
Fig. 5: Integrated link distribution for Gnguo/Fuego/Human

Can we have more information about network structure ?

Is there any difference between human and computer network ?

III Google Matrix and PageRanking

- $H_{ij} = \begin{cases} \frac{1}{l_j} \; si \; P_j \in B_i \\ 0 \quad sinon \end{cases}$ => $S_{ij} = \begin{cases} \frac{1}{N_V} \; si \; \forall i \; H_{ik} = 0 \\ H_{ij} \; sinon \end{cases}$



- $G_{ij} = \alpha S_{ij} + (1 - \alpha) \frac{1}{N}$

$$\begin{pmatrix} 0 & 0 & 1/8 & 0 & 1/2 & 0 & 1/8 & 0 \\ 0 & 0 & 1/8 & 0 & 0 & 1/3 & 1/8 & 0 \\ 0 & 0 & 1/8 & 0 & 0 & 1/3 & 1/8 & 0 \\ 0 & 0 & 1/8 & 0 & 0 & 0 & 1/8 & 1/2 \\ 1 & 1 & 1/8 & 0 & 0 & 0 & 1/8 & 0 \\ 0 & 0 & 1/8 & 0 & 1/2 & 0 & 1/8 & 0 \\ 0 & 0 & 1/8 & 0 & 0 & 1/3 & 1/8 & 1/2 \\ 0 & 0 & 1/8 & 1 & 0 & 0 & 1/8 & 0 \end{pmatrix}$$

Fig. 6: S_{ij}

$$\begin{pmatrix} 3/160 & 3/160 & 1/8 & 3/160 & 139/320 & 3/160 & 1/8 & 0 \\ 3/160 & 3/160 & 1/8 & 3/160 & 3/160 & 139/480 & 1/8 & 0 \\ 3/160 & 3/160 & 1/8 & 3/160 & 3/160 & 139/480 & 1/8 & 0 \\ 3/160 & 3/160 & 1/8 & 3/160 & 3/160 & 3/160 & 1/8 & 1/2 \\ 139/160 & 139/160 & 1/8 & 3/160 & 3/160 & 3/160 & 1/8 & 0 \\ 3/160 & 3/160 & 1/8 & 3/160 & 139/320 & 3/160 & 1/8 & 0 \\ 3/160 & 3/160 & 1/8 & 3/160 & 3/160 & 139/480 & 1/8 & 1/2 \\ 3/160 & 3/160 & 1/8 & 139/160 & 3/160 & 3/160 & 1/8 & 0 \end{pmatrix}$$

Fig. 7: G_{ij}

H : *Hyperlinks Matrix*
S : *Left Stochastic Matrix (eachc column summing to 1)*
G : *Google Matrix*

l_j: *number of outgoing link from* P_j
B_j: *ensemble of nodes with outgoing links toward j*
α : *damping factor*

3.1 Perron's Vector

- Let A be a real, non-negative and asymmetric Matrix with each column summing to 1 => every eigenvalues are less or equal in absolute value to 1
- The perron vector is the leading eigenvector associated with $\lambda = 1$

  Such that : $Ap = p$
- This vector represents the asymptotic time a random walker spend in each node of the network

3.2 Google PageRanking

- Using Perron-Frobenius theorem and iterative method => Ranking Indexed webpage by importance order
- A node i is more important if it is pointing by importants nodes…
- The importance of a node is proportional to its value within google matrix perron’s vector
- Damping factor turns $\mathbb{S}$ into a diagonalisable matrix with no degenerated leading eigenvalue

$\alpha \in [0, 1]$ *, we will take the value* 0.85

3.3 PageRank

- p is a perron's vector with size N

  We call P, the PageRank with $1 \leq P_k \leq N$

  as the permutation of integers obtained by ranking in decreasing order according to the entries p_i of the perron's vector

  such that => $p_{P(1)} \geq p_{P(2)} \geq \ldots \geq p_{P(N)}$

3.4 Other Ranking Vector

- CheiRank is the PageRank applied to the reciprocal network inverting incoming and outgoing links
- We can also use the other eigenvectors => information about different communities of nodes in the network

IV Distinguishing Human from Computer Strategy

- In the game of go case, our directed network reveals the strategy used within a databank of games
- The associated PageRank lights us about the most important moves played within the databank
- Let see here how can most important moves and eigenvalues for each network be involved in a Turing-like test => Is the computer imitate a human player ?

4.1 Top20

Gnugo

Fuego

Human

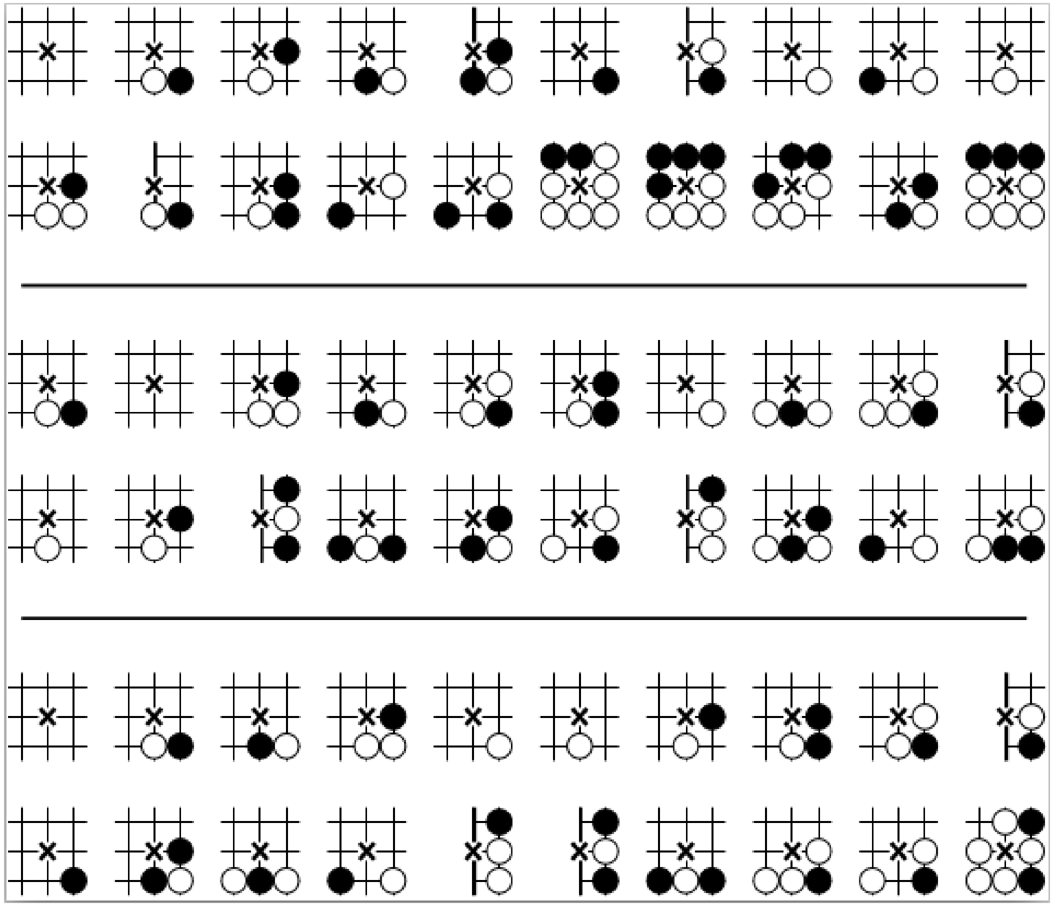


Fig. 8: *Top 20 of the PageRank, for Gnugo only 12 elements are in Human Top20 and for Fuego 18 elements are in Human Top20*

4.2 PageRank Correlation

- We plot the correlation between the first half of the entries for two PageRanks A and B
- In order to quantify we compute σ the dispersion

$$\sigma(A,B) = \left(\frac{\sum_{k=1}^{\lfloor N/2 \rfloor} (a_k - b_k)^2}{\lfloor N/2 \rfloor} \right)^{1/2}$$

Human/Gnugo PageRank Correlation plots have a high dispersion value (193.48) contrary to Gnugo/Gnugo (24.04) and Human/Human (43.66)

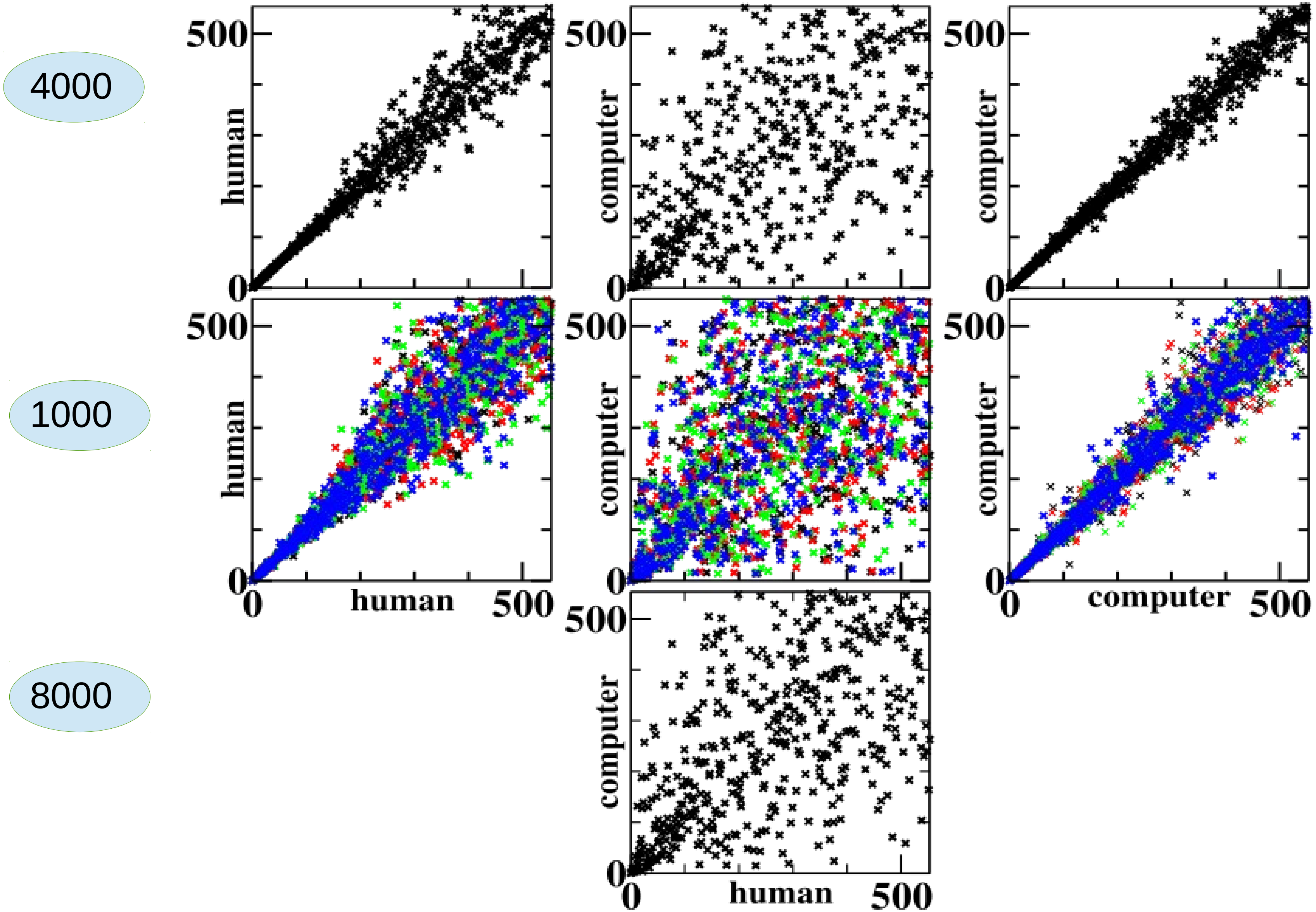


Fig. 9: PageRank *correlation, first column = human vs. Human, second column = computer vs. Human and last column = computer vs. computer*

4.3 Spectrum of Google Matrix

- Spectrum using 8000 games and $\alpha = 1$:

  Google matrix properties :

  the eigenvalues lie inside the unit disk

  complex ones occur in conjugated pairs

  Gnugo => scattered spectrum

  Fuego => similarity with Human but there are many outlying eigenvalues

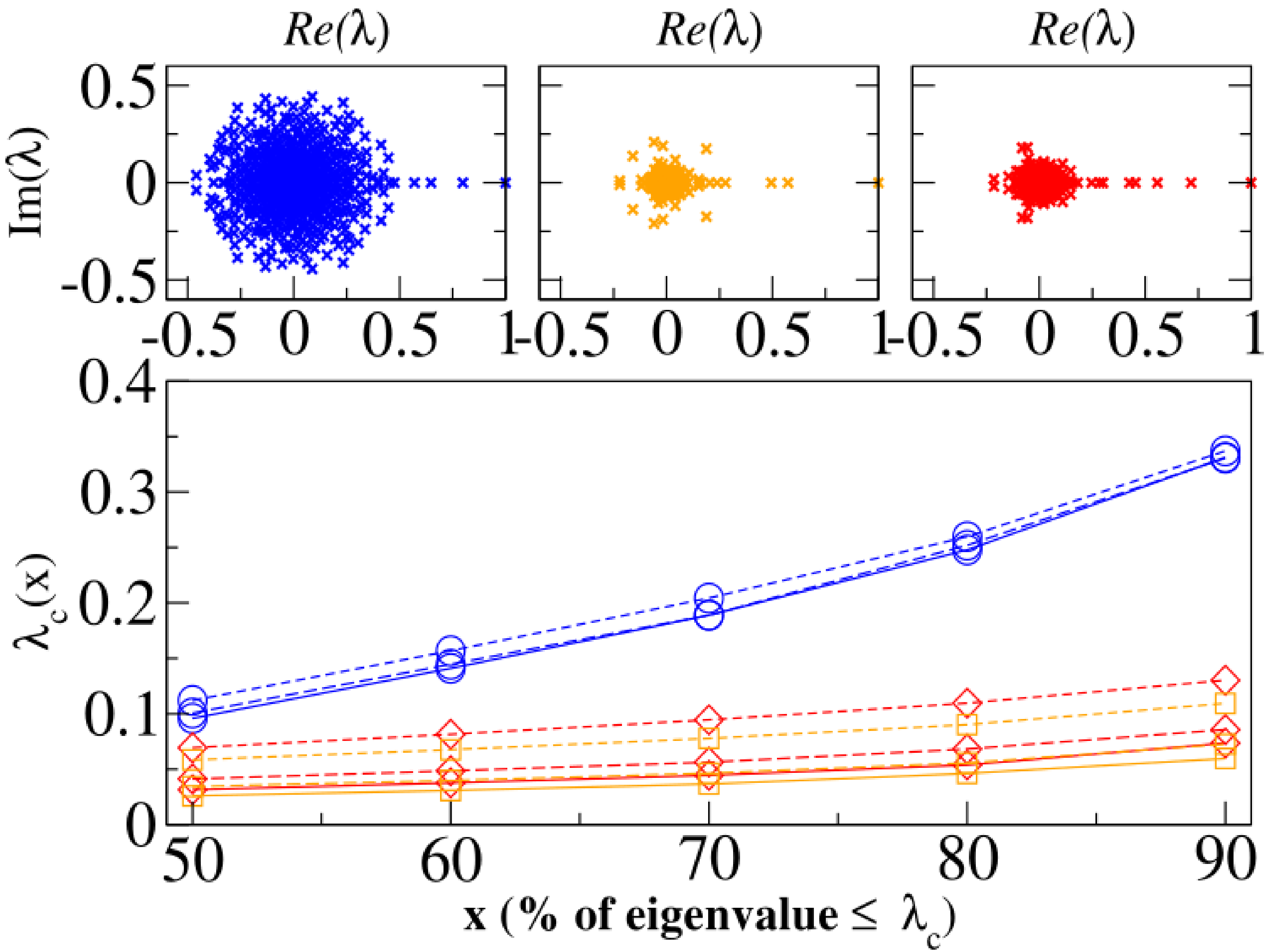
- $\lambda_c(x)$ = radius of a circle (centred at 0) containing a certain percentage x of eigenvalues

  => more quantitative informations

  => striking differences between the two behaviours

  => robust results with subset size

Fig. 10: Spectrum of the Google matrice for Gnugo, Fuego and Human with 8000-games network and $\alpha = 1.0$ (top row). Radial distribution if eigenvalues (bottom) same color code, solid line for 8000, long dashed 4000 and dashed 1000 games



4.4 Distinguishing with PageRank

- We want to compare PageRanks from different networks

  240 sets of 1000-games network, 120 of 2000-games network and 60 of 4000-games network

  1 master group of 8000 games

  different quantities : PageRank Fidelity F and PageRank Non-ordered PageRank Similarity S_N

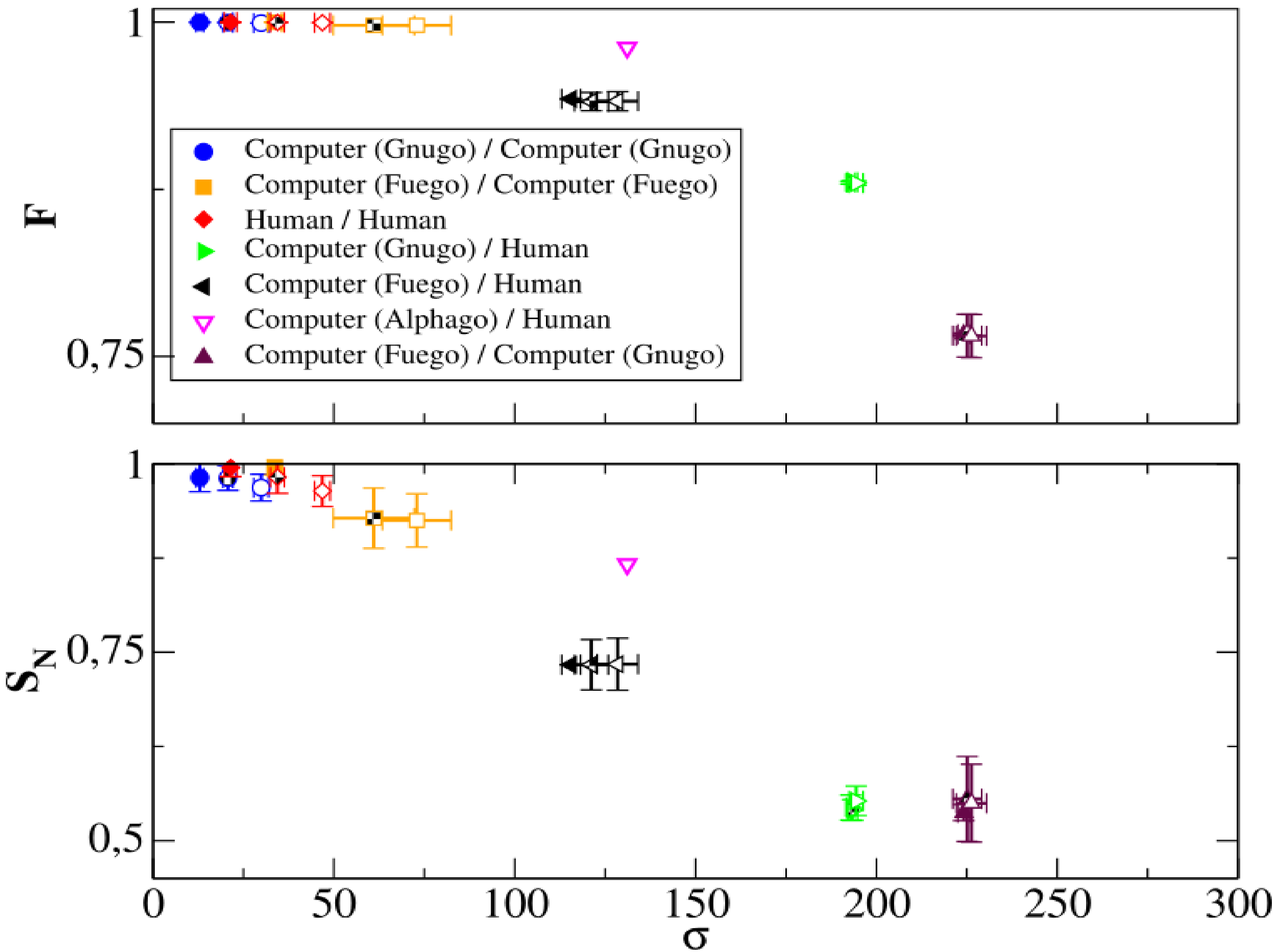
$$F = \left| \sum_{i=1}^{N} \phi_i^* \psi_i \right|$$

Perron's vectors normalized using a norme-2 condition
$F = 0$ => totaly different
$F = 1$ => same vectors

$$S_N(A, B) = \sum_{i=1}^{30} \frac{f_{bis}(i)}{30}$$

$$f_{bis}(i) = \begin{cases} 1 & \text{if } \exists j \in [1;30] \text{ such that } a_i = b_j \\ 0 & \text{otherwise.} \end{cases}$$

4.5 Toward a Turing-like Test for go



Conclusion

- Networks built from computer-generated games and human-played games have statistically significant differences in several respects :
  - Google matrix Spectra
  - PageRank vector
- Differences using different algorithms
  - Deterministic (Gnugo), Monte-Carlo (Fuego) and even with a small database with deep learning simulator (AlphaGo)

=> In general the computer plays using more varied set of most played moves, but with more correlation between games for Gnugo

=> We could devise a Turing test for the go simulators

Does this simulator imitate very well a biological palyer ?

Thank you