Modeling the rotation of Mercury

Benoît Noyelles

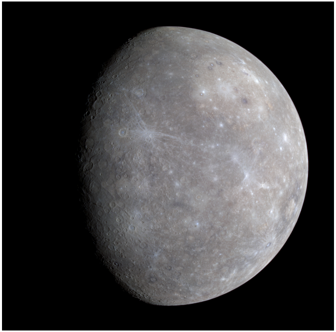
University of Namur (Belgium)
in collaboration with A. Lemaître, C. Lhotka, J. Frouard, S. D'Hoedt and J. Dufey

May, 17^{th} 2013 - Rio Claro

Plan

Plan

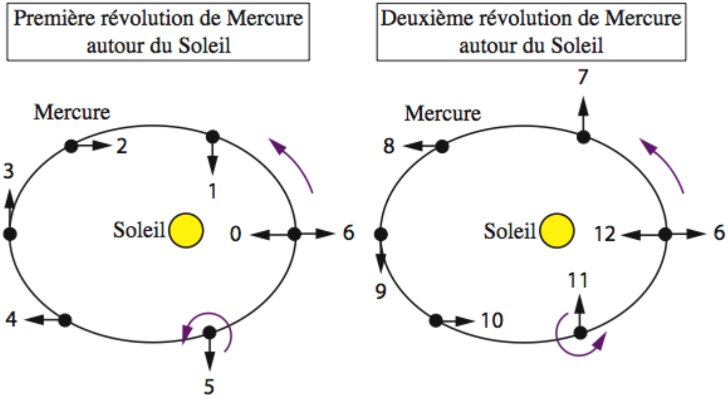
The dynamics of Mercury



- Semi-major axis: 0.389 AU
- Eccentricity: 0.206
- Inclination: 7°
- Orbital period: 88 days
- Spin period: 58 days
- Obliquity: 2.04 ± 0.08 arcmin

Radius: 2439.7 ± 1.0 km

The rotation of Mercury



3:2 spin-orbit resonance

The mission MESSENGER (NASA)

Mercury Surface, Space Environment, Geochemistry and Ranging

- Launched on August 3, 2004
- 3 flybys : January 14 2008, October 6 2008 and September 29 2009
- Orbit insertion : March 18, 2011
- Goals : Maps, 3-D model of magnetosphere, gravity field, etc.

The gravity field of Mercury

$$U(r,\lambda,\phi) = \frac{GM}{r} + \frac{GM}{r}\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^{n} P_{nm}(\sin\phi)\left[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right]$$

r: distance, λ: longitude, ϕ: latitude

	Mariner 10 (1973)	2 flybys (2010)	MESSENGER (2012)
$C_{20} = -J_2$	$(-6.0 \pm 2.0) \times 10^{-5}$	$(-1.92 \pm 0.67) \times 10^{-5}$	$(-5.031 \pm 0.02) \times 10^{-5}$
C_{21}	–	–	$(-5.99 \pm 6.5) \times 10^{-8}$
S_{21}	–	–	$(1.74 \pm 6.5) \times 10^{-8}$
C_{22}	$(1.0 \pm 0.5) \times 10^{-5}$	$(8.1 \pm 0.8) \times 10^{-6}$	$(8.088 \pm 0.065) \times 10^{-6}$
S_{22}	–	$(-0.3 \pm 1.2) \times 10^{-6}$	$(3.22 \pm 6.5) \times 10^{-8}$
$C_{30} = -J_3$	–	–	$(-1.188 \pm 0.08) \times 10^{-5}$
$C_{40} = -J_4$	–	–	$(-1.95 \pm 0.24) \times 10^{-5}$

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The gravity field of Mercury
Smith et al. 2012

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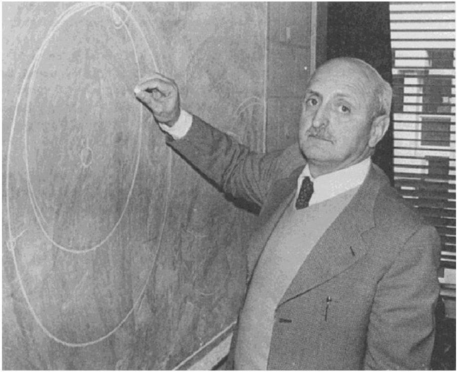
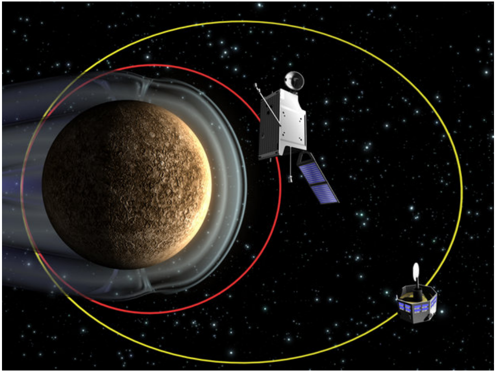
The mission BepiColombo (1/3)
(ESA / JAXA)



Giuseppe Colombo (1920-1984)

The mission BepiColombo (2/3)



Launch : August 2015

The mission BepiColombo (3/3)

MPO (Mercury Planetary Orbiter, ESA)

- 11 instruments
- 400 - 1500 km
- Period: 2.3 h

MMO (Mercury Magnetospheric Orbiter, JAXA)

- 5 instruments
- 400 - 11800 km
- Period: 9.3 h

The MORE experiment

Mercury Orbiter Radio-Science Experiment

Goals

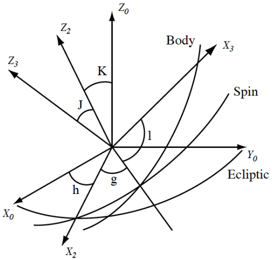
- Gravity field of Mercury
- Test of General Relativity (PPN γ, β, η, α_1, + Solar J_2)
- Rotation

Our job in Namur

Provide a routine giving the matrix between an inertial reference frame and the principal axes of inertia of Mercury at any date and for any set of relevant interior parameters (C_{20}, C_{22}, size of the core, ...)

Plan

The variables
3 degrees of freedom



- $\lambda_1 = l + g + h$
- $\lambda_2 = -l$
- $\lambda_3 = -h$

- Λ_1 (Angular Momentum)
- $\Lambda_2 = \Lambda_1(1 - \cos J)$ (Wobble)
- $\Lambda_3 = \Lambda_1(1 - \cos K)$ (Obliquity)

(from D'Hoedt & Lemaître 2004)

The Hamiltonian

$$\begin{aligned}\mathcal{H} &= \frac{(\Lambda_1-\Lambda_2)^2}{2C} + \frac{\Lambda_1^2-(\Lambda_1-\Lambda_2)^2}{2}\left(\frac{\sin^2\lambda_2}{A}+\frac{\cos^2\lambda_2}{B}\right) \\ &- \frac{3}{2}\frac{GM_\odot}{r^3}MR^2\left(J_2(x^2+y^2)+C_{22}(x^2-y^2)\right)\end{aligned}$$

with :

- $M_\odot$: Solar mass
- (x, y, z): unit vector pointing at the Sun in the Hermean frame
- $A < B < C$: principal moments of inertia
- $J_2 = \frac{2C-B-A}{2MR^2}$, $C_{22} = \frac{B-A}{4MR^2}$

The equilibrium

2 “resonant” arguments

- $\sigma_1 = \lambda_1 - \frac{3}{2}l_o - \varpi_o$ (spin-orbit resonance)
- $\sigma_3 = \lambda_3 + \Omega_o$ (3rd Cassini Law)

The equilibrium

- $\Lambda_1^* = \frac{3}{2}nC$ (rigid)
- $\sigma_1^* = 0$, $\sigma_3^* = 0$
- $J^* = 0$, $K^* \propto \dot{\Omega}_o$

The free librations (1/2)

The method

Centering the Hamiltonian

$$\begin{aligned}\mathcal{H}_1 &= \alpha_1\sigma_1^2 + 2\alpha_2\sigma_1\sigma_3 + \alpha_3\sigma_3^2 \\ &+ \alpha_4\eta_1^2 + 2\alpha_5\eta_1\eta_3 + \alpha_6\eta_3^2 \\ &+ \alpha_7\lambda_2^2 + \alpha_8\Lambda_2^2 + \text{third-order}\end{aligned}$$

Obtention of the free librations

- Untangling the degrees of freedom (Henrard & Lemaître 2005)
- We get : $\mathcal{H}_2 = \omega_u U + \omega_v V + \omega_w W$

The free librations (2/2)
Numerical values

	Rigid case	Liquid core ($C_m/C = 0.579$)
T_u	15.85 y	12.06 y
T_v	1065 y	616 y
T_w	582 y	460 y

3 timescales of excitation. . .
(at least)

- Orbital period of Mercury: 88 days
- Orbital period of Jupiter: 11.86 years
- Nodal regression period: $\approx$ 300 kyrs

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The Peale experiment

Question

Does Mercury have a partially molten core?

Assumptions

- For short-term motion (longitudinal librations), the (spherical) fluid core does not affect the rotation
- For long-term motion (obliquity), the fluid core behaves like a rigid body

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The equations

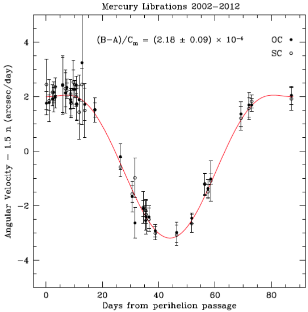
Amplitude of the longitudinal librations (88 days)

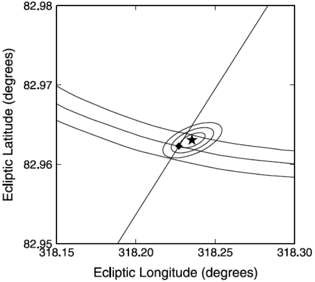
$$\begin{aligned}\phi &= \frac{3}{2}\frac{B-A}{C_m}\left(1-11e^2+\frac{959}{48}e^4+\ldots\right)\\ &= 6C_{22}\frac{MR^2}{C}\frac{C}{C_m}\left(1-11e^2+\frac{959}{48}e^4+\ldots\right)\end{aligned}$$

Equilibrium obliquity (Cassini State 1)

$$\epsilon = -\frac{\frac{C}{MR^2}\dot{\Omega}\sin\iota}{\frac{C}{MR^2}\dot{\Omega}\cos\iota + 2n\left(\frac{7}{2}e-\frac{123}{16}e^3\right)C_{22} - n\left(1-e^2\right)^{-3/2}C_{20}}$$

The observed quantities
Margot et al. 2007, 2012



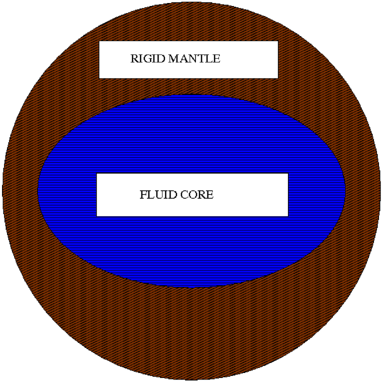


$\phi = (38.5 \pm 1.6)$ arcsec $\epsilon = (2.04 \pm 0.08)$ arcmin

$$C_m/C = 0.431 \pm 0.025 \qquad C/(MR^2) = 0.346 \pm 0.014$$

Including the pressure coupling

the core is now triaxial



$$
\begin{aligned}
\epsilon_1 &= \frac{2C - A - B}{2C} = J_2 \frac{MR^2}{C} \\
\epsilon_2 &= \frac{B - A}{2C} = 2C_{22} \frac{MR^2}{C} \\
\epsilon_3 &= \frac{2C_c - A_c - B_c}{2C_c} \\
\epsilon_4 &= \frac{B_c - A_c}{2C_c} \\
\delta &= C_c / C
\end{aligned}
$$

The variables of the problem

4 degrees of freedom

Canonical variables

- (p, P): spin & norm of the total angular momentum
- (r, R): node & obliquity
- (η_1, ξ_1): polar momentum of the whole body
- $(\eta_2, \xi_2) \approx$ velocity field of the fluid

Interesting quantities

- Longitudinal librations ϕ_m
- Obliquity of the mantle K_m
- Polar motion of the mantle J_m
- Tilt of the velocity field of the fluid J_c

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The Hamiltonian of the model

Noyelles et al. 2010

$$
\begin{aligned}
\mathcal{H} \approx\ & \frac{n}{2(1-\delta)}\left(P^2+\frac{P_c^2}{\delta}+2\sqrt{PP_c}(\eta_1\eta_2-\xi_1\xi_2)+2\left(P\frac{\xi_2^2+\eta_2^2}{2}+P_c\frac{\xi_1^2+\eta_1^2}{2}-PP_c\right)\right) \\
+\ & \frac{n\epsilon_1}{2(1-\delta)^2}\left(P(\xi_1^2+\eta_1^2)+P_c(\xi_2^2+\eta_2^2)+2\sqrt{PP_c}(\eta_1\eta_2-\xi_1\xi_2)\right) \\
+\ & \frac{n\epsilon_2}{2(1-\delta)^2}\left(P(\xi_1^2-\eta_1^2)+P_c(\xi_2^2-\eta_2^2)-2\sqrt{PP_c}(\eta_1\eta_2+\xi_1\xi_2)\right) \\
-\ & \frac{n\epsilon_3}{2(1-\delta)^2}\left(\delta P(\xi_1^2+\eta_1^2)+\left(2-\frac{1}{\delta}\right)P_c(\xi_2^2+\eta_2^2)+2\delta\sqrt{PP_c}(\eta_1\eta_2-\xi_1\xi_2)\right) \\
+\ & \frac{n\epsilon_4}{2(1-\delta)^2}\left(\delta P(\eta_1^2-\xi_1^2)+\left(2-\frac{1}{\delta}\right)P_c(\eta_2^2-\xi_2^2)+2\delta\sqrt{PP_c}(\eta_1\eta_2+\xi_1\xi_2)\right) \\
-\ & \frac{3}{2}\frac{\mathcal{G}M}{nr^3}\left(\epsilon_1(x^2+y^2)+\epsilon_2(x^2-y^2)\right)
\end{aligned}
$$

How the shape of the core affects the frequencies

shape $\equiv$ triaxiality

	ϵ_3/ϵ_1	0	0.1	1	3	3
	ϵ_4/ϵ_2	0	0	1	3	0
(longitude)	T_u (y)	12.05800	12.05775	12.05772	12.05777	12.05773
(obliquity)	T_v (y)	615.77	(large)	1636.43	1214.91	1216.09
(wobble)	T_w (y)	337.82	337.82	337.87	338.14	338.20
(Free Core Nutation)	T_z (d)	–	58.630	58.619	58.585	58.585
	$T_{z-\omega}$ (y)	–	574.06	343.45	154.04	154.01

ϵ_3, ϵ_4: polar flattening, equatorial ellipticity of the core

The equatorial flattening (ϵ_4) of the core has no influence on the rotational dynamics.

The longitudinal librations are well represented with a spherical core.

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Longitudinal librations

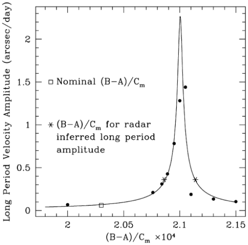
Dufey et al. (2008, 2009)

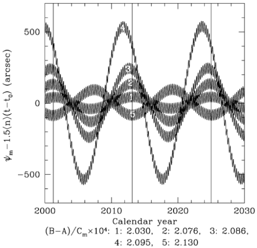
Free period : 12.06 years

N	l_o Mercury	l_v Venus	l_e Earth	l_j Jupiter	l_s Saturn	Period	Amplitude	Ratio
1	-	-	-	1	-	11.862 y	43.711 as	1.2193
2	1	-	-	-	-	87.970 d	35.848 as	1.0000
3	2	-	-	-	-	43.985 d	3.754 as	0.1047
4	2	-5	-	-	-	5.664 y	3.597 as	0.1003
5	-	-	-	-	2	14.729 y	1.568 as	0.0437
6	-	-	-	2	-	5.931 y	1.379 as	0.0385
7	1	-	-4	-	-	6.575 y	0.578 as	0.0161
8	3	-	-	-	-	29.323 d	0.386 as	0.0108
9	1	-	-	-2	-	91.692 d	0.201 as	0.0056
10	1	-	-	2	-	84.537 d	0.191 as	0.0053
11	-	-	-	2	-5	883.28 y	0.103 as	0.0029
12	2	-	-	-1	-	44.436 d	0.069 as	0.0019
13	2	-	-	1	-	43.541 d	0.067 as	0.0019
14	1	-	-	-1	-	89.793 d	0.044 as	0.0012
15	1	-	-	1	-	86.217 d	0.043 as	0.0012
16	2	-	-	-2	-	44.897 d	0.041 as	0.0011
17	2	-	-	2	-	43.110 d	0.040 as	0.0011

A longitudinal resonance

- Free period: 12.06 years
- Orbital period of Jupiter: 11.86 years

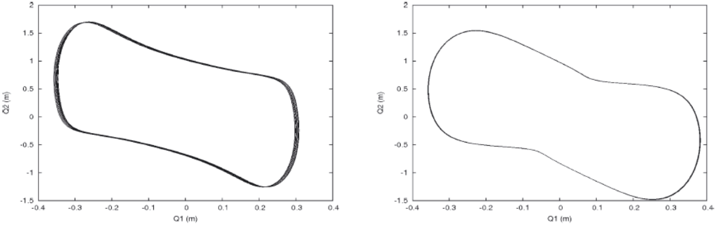




(Peale, Margot & Yseboodt 2009)

The forced polar motion

cannot be detected. . .



$\epsilon_3/\epsilon_1 = \epsilon_4/\epsilon_2 = 1$ $\quad$ $\epsilon_3 = 3\epsilon_1, \epsilon_4 = 0$

Period: 175.9 days

This motion is negligible.

Plan

Our job for MORE / BepiColombo

Our job

Provide a routine giving the matrix between an inertial reference frame and the principal axes of inertia of Mercury at any date and for any set of relevant interior parameters (C_{20}, C_{22}, size of the core, ...)

Problem

It is very difficult to simulate the rotation without generating free librations.

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The way we proceed

- Consideration of 2 degrees of freedom (polar motion neglected)
- Numerical integrations of the equations of motion over 6000 yrs
- Use of JPL/DE406 ephemerides (available over -3000 +3000)
- Damping of the free longitudinal librations (period: $\approx$ 12 yrs)
- Problem: How to remove the free librations of the obliquity? (period: $\approx$ 1 kyr)

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Minimizing the free librations in obliquity

Problem

Difficult to damp librations of 1000 yrs over 4000 yrs without altering significantly the equilibrium.

Idea

Optimize the initial conditions.

Algorithm: a long-term study

- Averaging of the equations of motion
- Extrapolation of the relevant orbital quantities
- Frequency analysis to remove the free librations
- Expression of the initial conditions

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Extrapolation of the eccentricity
Noyelles & D'Hoedt 2012

$$z(t) = e(t)\exp\imath\varpi(t) = e(t)\left(\cos\varpi(t) + \imath\sin\varpi(t)\right) = k(t) + \imath h(t)$$

$h(t) = e(t)\sin\varpi(t)$

$$\begin{aligned}
h(t) &\approx a_2 t^2 + a_1 t + a_0 \\
&\approx \alpha_1 \sin(\dot\omega_1 t + \phi_1) + \alpha_2 \sin(\dot\omega_2 t + \phi_2) \\
a_0 &= \alpha_1 \sin\phi_1 + \alpha_2 \sin\phi_2 \\
a_1 &= \alpha_1\dot\omega_1 \cos\phi_1 + \alpha_2\dot\omega_2 \cos\phi_2 \\
a_2 &= -\left(\alpha_1\dot\omega_1^2 \sin\phi_1 + \alpha_2\dot\omega_2^2 \sin\phi_2\right)/2
\end{aligned}$$

$k(t) = e(t)\cos\varpi(t)$

$$\begin{aligned}
k(t) &\approx a_5 t^2 + a_4 t + a_3 \\
&\approx \alpha_1 \cos(\dot\omega_1 t + \phi_1) + \alpha_2 \cos(\dot\omega_2 t + \phi_2) \\
a_3 &= \alpha_1 \cos\phi_1 + \alpha_2 \cos\phi_2 \\
a_4 &= -\left(\alpha_1\dot\omega_1 \sin\phi_1 + \alpha_2\dot\omega_2 \sin\phi_2\right) \\
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\end{aligned}$$

Extrapolation of the inclination

$$\zeta(t) = \sin\frac{I(t)}{2}\exp\imath\Omega(t) = \sin\frac{I(t)}{2}\left(\cos\Omega(t) + \imath\sin\Omega(t)\right) = q(t) + \imath p(t)$$

$p(t) = \sin\frac{I(t)}{2}\sin\Omega(t)$

$$\begin{aligned}
p(t) &\approx b_2 t^2 + b_1 t + b_0 \\
&\approx \beta_1 \sin(\dot{\Omega}_1 t + \Phi_1) + \beta_2 \sin(\dot{\Omega}_2 t + \Phi_2) \\
b_0 &= \beta_1 \sin\Phi_1 + \beta_2 \sin\Phi_2 \\
b_1 &= \beta_1 \dot{\Omega}_1 \cos\Phi_1 + \beta_2 \dot{\Omega}_2 \cos\Phi_2 \\
b_2 &= -\left(\beta_1 \dot{\Omega}_1^2 \sin\Phi_1 + \beta_2 \dot{\Omega}_2^2 \sin\Phi_2\right)/2
\end{aligned}$$

$q(t) = \sin\frac{I(t)}{2}\cos\Omega(t)$

$$\begin{aligned}
q(t) &\approx b_5 t^2 + b_4 t + b_3 \\
&\approx \beta_1 \cos(\dot{\Omega}_1 t + \Phi_1) + \beta_2 \cos(\dot{\Omega}_2 t + \Phi_2) \\
b_3 &= \beta_1 \cos\Phi_1 + \beta_2 \cos\Phi_2 \\
b_4 &= -\left(\beta_1 \dot{\Omega}_1 \sin\Phi_1 + \beta_2 \dot{\Omega}_2 \sin\Phi_2\right) \\
b_5 &= -\left(\beta_1 \dot{\Omega}_1^2 \cos\Phi_1 + \beta_2 \dot{\Omega}_2^2 \cos\Phi_2\right)/2
\end{aligned}$$

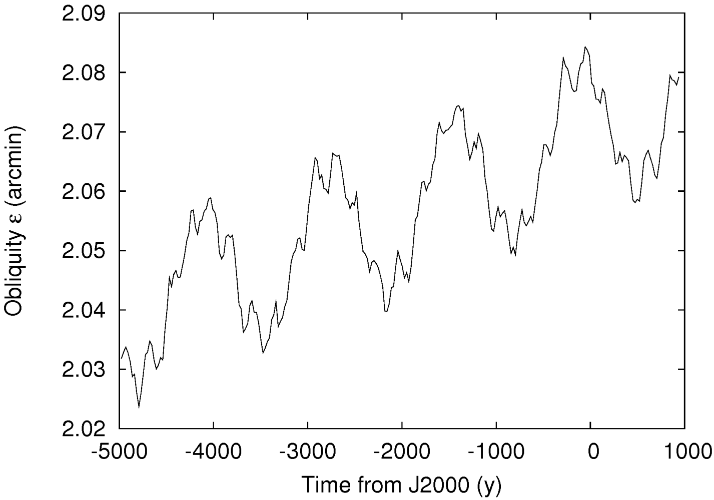
New initial conditions

$$
\begin{aligned}
K &= I + a_1 - 2a_2\cos(\Omega_2-\Omega_1) + 2a_3\cos(2\Omega_2-2\Omega_1) - 2a_4\cos(\varpi_1-\varpi_2) \\
&+ 2a_5\cos(2\varpi_1-2\Omega_1) - 2a_6\cos(3\Omega_2-3\Omega_1) + 2a_7\cos(4\Omega_2-4\Omega_1) \\
&+ 2a_8\cos(\varpi_2-\varpi_1+\Omega_2-\Omega_1) + 2a_9\cos(\varpi_1-\varpi_2+\Omega_2-\Omega_1) + 2a_{10}\cos(\varpi_1+\varpi_2-2\Omega_1) \\
&- 2a_{11}\cos(2\varpi_1-3\Omega_1+\Omega_2) - 2a_{12}\cos(5\Omega_2-5\Omega_1) + 2a_{13}\cos(2\varpi_1-2\varpi_2) \\
&- 2a_{14}\cos(\varpi_1-\varpi_2-2\Omega_1+2\Omega_2) - 2a_{15}\cos(\varpi_2-\varpi_1-2\Omega_1+2\Omega_2) \\
&- 2a_{16}\cos(2\varpi_1-2\Omega_2)\,, \\
\sigma_3 &= 2a_{17}\sin(\Omega_2-\Omega_1) - 2a_{18}\sin(2\Omega_2-2\Omega_1) + 2a_{19}\sin(3\Omega_2-3\Omega_1) \\
&- 2a_{20}\sin(2\varpi_1-2\Omega_1) - 2a_{21}\sin(4\Omega_2-4\Omega_1) - 2a_{22}\sin(\varpi_2-\varpi_1+\Omega_2-\Omega_1) \\
&- 2a_{23}\sin(\varpi_1-\varpi_2+\Omega_2-\Omega_1) + 2a_{24}\sin(2\varpi_1-3\Omega_1+\Omega_2) + 2a_{25}\sin(5\Omega_2-5\Omega_1) \\
&+ 2a_{26}\sin(2\varpi_1-\Omega_1-\Omega_2) - 2a_{27}\sin(\varpi_1+\varpi_2-2\Omega_1) + 2a_{28}\sin(-\varpi_1+\varpi_2-2\Omega_1+2\Omega_2) \\
&+ 2a_{29}\sin(\varpi_1-\varpi_2-2\Omega_1+2\Omega_2) - 2a_{30}\sin(2\varpi_1-4\Omega_1+2\Omega_2) - 2a_{31}\sin(6\Omega_2-6\Omega_1) \\
&+ 2a_{32}\sin(\varpi_1+\varpi_2-3\Omega_1+\Omega_2) - 2a_{33}\sin(\varpi_1-\varpi_2-3\Omega_1+3\Omega_2) \\
&- 2a_{34}\sin(\varpi_2-\varpi_1-3\Omega_1+3\Omega_2)\,,
\end{aligned}
$$

with

$$a_i = \frac{C/\left(MR^2\right)}{\alpha_i C/\left(MR^2\right) + \beta_i C_{20} + \gamma_i C_{22} + \delta_i}$$

An example of resulting obliquity



Checking Peale's formula
Noyelles & Lhotka 2013

From obliquity $\epsilon = (2.04 \pm 0.08)$ arcmin

- Margot et al.(2012): $C/(MR^2) = 0.346 \pm 0.014$
- Analytical formula: $C/(MR^2) = 0.34712 \pm 0.01361$
- Numerical formula: $C/(MR^2) = 0.34576 \pm 0.01349$
- Analytical formula + J_3: $C/(MR^2) = 0.34640 \pm 0.01361$
- Numerical formula + J_3: $C/(MR^2) = 0.34506 \pm 0.01348$

The influence of usually neglected effects

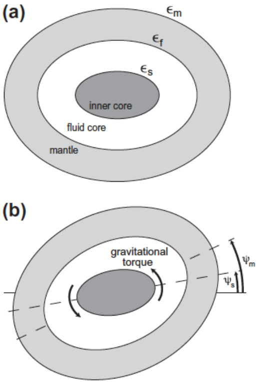
Effect	Influence on obliquity
Free librations	$<$ 750 mas
C_{30}	$\approx$ 250 mas
Polar motion	$\approx$ 80 mas
Tides	$\approx$ 30 mas
Short-period librations	$<$ 20 mas
Secular drift	$\approx$ 10 mas over 20 years
C_{40}	negligible

Plan

1. Introduction
2. Mathematical formulation of the rotation
   - A rigid Mercury
   - Equilibrium and free periods
   - Introduction of a liquid core
   - The Poincaré-Hough model
3. Our numerical treatment
4. Perspective and conclusions

A rigid inner core?

Veasey & Dumberry 2011,. . .



$$\ddot{\gamma}_m = -\frac{3}{2}\left(\frac{B_m - A_m}{C_m}\right)\left[\frac{1 + e\cos f}{1 - e^2}\right]^3 \sin(3t + 2\gamma_m - 2f) - \frac{\overline{\Gamma} a^3}{GM_\odot C_m}\sin 2(\gamma_m - \gamma_s),$$

$$\ddot{\gamma}_s = -\frac{3}{2}\alpha\left(\frac{B_s - A_s}{C_s}\right)\left[\frac{1 + e\cos f}{1 - e^2}\right]^3 \sin(3t + 2\gamma_s - 2f) + \frac{\overline{\Gamma} a^3}{GM_\odot C_s}\sin 2(\gamma_m - \gamma_s).$$

Conclusions

- We have exposed different aspects of the rotation of Mercury
- Different methods have been developed for this purpose
- We hope to get more clues on Mercury's interior
- Next step: an inner rigid core

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