Effect of pressure coupling on the rotation of Mercury

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We present an adaptation of the Poincaré model of core-mantle interaction to Mercury, seen as composed of a rigid mantle and an elliptical liquid core. Thanks to a Hamiltonian formulation, we perform extensively both an analytical (Lie transforms) and a numerical analysis of this 4-degree of freedom problem. This allows us to highlight a long-term behavior of the obliquity, the consequences of the proximity of a resonance between the spin frequency of Mercury and the free core nutation, and also the influence of the polar flattening of the core.

Modeling the interior of Mercury

In the framework of the ESA space mission Bepi-Colombo, planned to be launched in 2014 and to reach Mercury 6 years later, we model the rotation of Mercury so that its observations could be inverted to get data on Mercury's gravity field. Radar observations of the longitudinal librations of Mercury suggest the existence of an at least molten core below a rigid mantle. In previous studies we assumed the mantle to be spherical, while we here consider it as elliptic (see Fig.1).

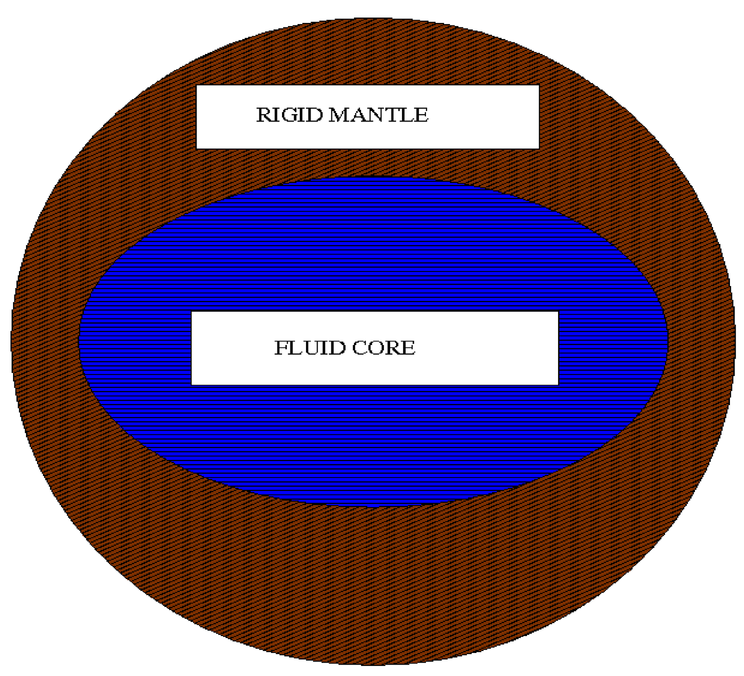

Figure 1: Our representation of Mercury's interior.

As a consequence, the fluid constituting the core exerts a pressure coupling at the core-mantle boundary. We use the model of Poincaré to study the dynamics of the system, that assumes the fluid to be inviscid with constant uniform density and vorticity. A simple velocity field inside the core is assumed, adding a 4th degree of freedom to a model of rigid rotation of the mantle. Moreover, we consider that the rotation of Mercury is perturbed by the Sun. Under these assumptions, the Hamiltonian of the system reads:

$$
\begin{aligned}
\mathcal{H} = & \frac{n}{2(1-\delta)}\Bigg(P^2+\frac{P_c^2}{\delta}+2\sqrt{PP_c}(\eta_1\eta_2-\xi_1\xi_2) \\
& +2\Big(P\frac{\xi_2^2+\eta_2^2}{2}+P_c\frac{\xi_1^2+\eta_1^2}{2}-PP_c\Big)\Bigg) \\
& +\frac{n\epsilon_1}{2(1-\delta)^2}\Bigg(P\big(\xi_1^2+\eta_1^2\big)+P_c\big(\xi_2^2+\eta_2^2\big)+2\sqrt{PP_c}(\eta_1\eta_2-\xi_1\xi_2)\Bigg) \\
& +\frac{n\epsilon_2}{2(1-\delta)^2}\Bigg(P\big(\xi_1^2-\eta_1^2\big)+P_c\big(\xi_2^2-\eta_2^2\big)-2\sqrt{PP_c}(\eta_1\eta_2+\xi_1\xi_2)\Bigg) \\
& -\frac{n\epsilon_3}{2(1-\delta)^2}\Bigg(\delta P\big(\xi_1^2+\eta_1^2\big)+\Big(2-\frac{1}{\delta}\Big)P_c\big(\xi_2^2+\eta_2^2\big) \\
& +2\delta\sqrt{PP_c}(\eta_1\eta_2-\xi_1\xi_2)\Bigg) \\
& +\frac{n\epsilon_4}{2(1-\delta)^2}\Bigg(\delta P\big(\eta_1^2-\xi_1^2\big)+\Big(2-\frac{1}{\delta}\Big)P_c\big(\eta_2^2-\xi_2^2\big) \\
& +2\delta\sqrt{PP_c}(\eta_1\eta_2+\xi_1\xi_2)\Bigg) \\
& -\frac{3\mathcal{G}M}{2\,nd^3}\big(\epsilon_1(x^2+y^2)+\epsilon_2(x^2-y^2)\big)
\end{aligned}
$$

with the following parameters characterizing the internal structure of Mercury:

$$
\begin{aligned}
\epsilon_1 &= \frac{2C-A-B}{2C}=J_2\frac{MR^2}{C}, \\
\epsilon_2 &= \frac{B-A}{2C}=2C_{22}\frac{MR^2}{C}, \\
\epsilon_3 &= \frac{2C_c-A_c-B_c}{2C_c}, \\
\epsilon_4 &= \frac{B_c-A_c}{2C_c}, \\
\delta &= C_c/C.
\end{aligned}
$$

ϵ_1, ϵ_2 and δ are assumed to be known, so we let the parameters ϵ_3 and ϵ_4 vary, i.e. the shape of the triaxial core.

Characterization of the problem

A 4-d.o.f. system can mathematically be characterized by 4 proper periods of free librations around the equilibrium, that is here the Cassini State at the 3 : 2 spin-orbit resonance. Mercury is assumed to be close to this equilibrium thanks to dissipative effects. Expressing the 4 proper periods is a way to represent the response of the system to external sollicitations (here the gravitational torque of the Sun). We calculated the numerical values of these periods both with a full numerical code and with Lie transforms with good agreement. The results are given in Tab.1:

Table 1: The proper periods of the system.

ϵ_3/ϵ_1	0	0.1	1	3	3
ϵ_4/ϵ_2	0	0	1	3	0
T_u (y)	12.05800	12.05775	12.05772	12.05777	12.05773
T_v (y)	615.77	(large)	1636.43	1214.91	1216.09
T_w (y)	337.82	337.82	337.87	338.14	338.20
T_z (d)	–	58.630	58.619	58.585	58.585

These periods have roughly these physical meanings :

- T_u: period of the free longitudinal librations,
- T_v: period of the free librations of the obliquity of Mercury,
- T_w: period of the free polar motion of Mercury,
- T_z: period of the free oscillations of the core.

We can see from their numerical values that the period T_z, associated with the core, is close to a resonance with the spin frequency of Mercury. This induces a slow convergence of the algorithm of Lie transforms, and dynamical effects that should raise the response of Mercury to 58-d periodic excitations. We see that the system is closer to the resonance when the polar flattening of the core ϵ_3 is small. We can also see that the periods T_u associated with longitudinal librations is quite constant, this induces that the shape of the core cannot be detected in the longitudinal librations of Mercury. Finally, the periods of the free librations in obliquity depends also on ϵ_3 as be can also seen on Fig.2. When ϵ_3 increases, T_v tends to the rigid value of 1,070 years.

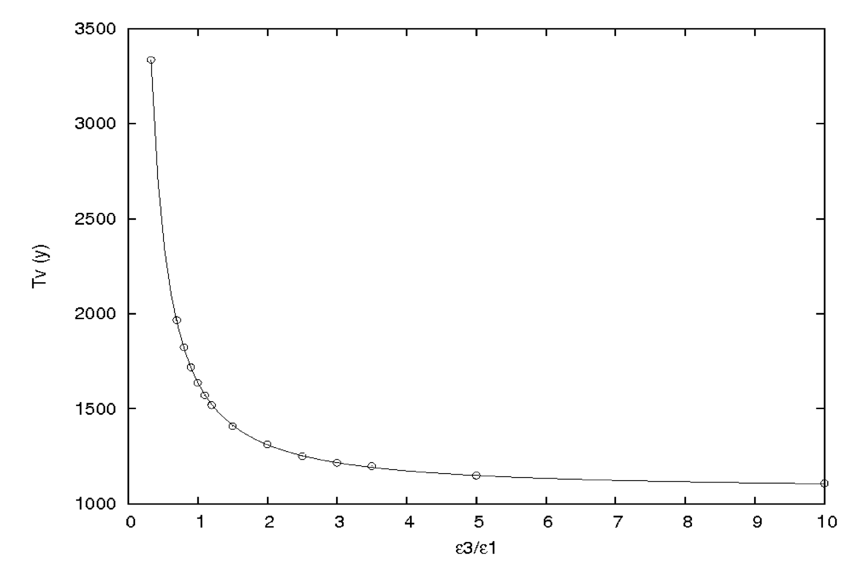

Figure 2: Evolution of the proper period T_v, associated with the obliquity. It seems to obey a law alike $T_v \propto \epsilon_3^{-5/4}$.

Results

We here express the observables outputs of the system, i.e. the orientation of the mantle (and not of Mercury). As predicted from the variations of T_u, the longitudinal librations are independent of the shape of the core if we assume its size as known. We here plot the obliquity of the mantle (Fig. 3), the polar motion (Fig. 4), and the wobble of the core (Fig.5).

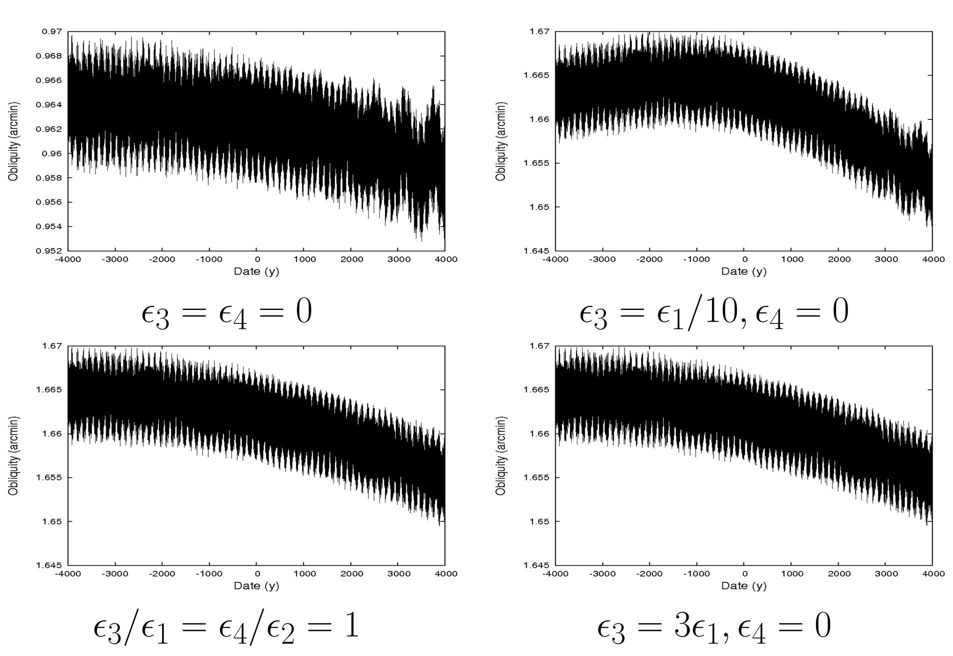

Figure 3: Variations of the orbital obliquity ϵ of the mantle of Mercury for different shapes of the core. Two behaviors can be distinguished: when the core is spherical ($\epsilon_3=\epsilon_4=0$), and when it is not.

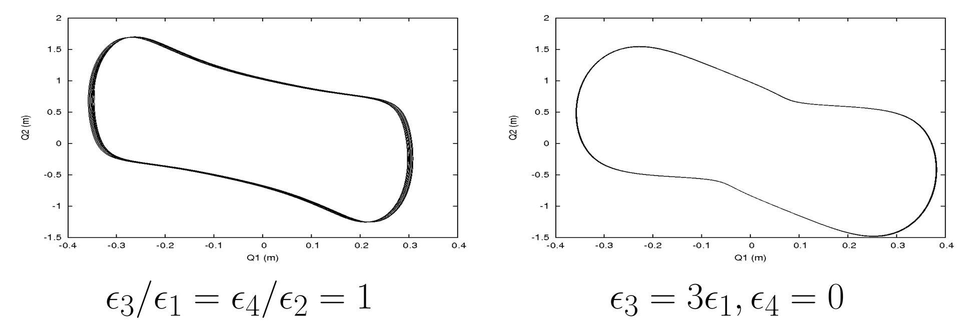

Figure 4: Polar motion of the mantle. It is very small and quite independent of the shape of the core.

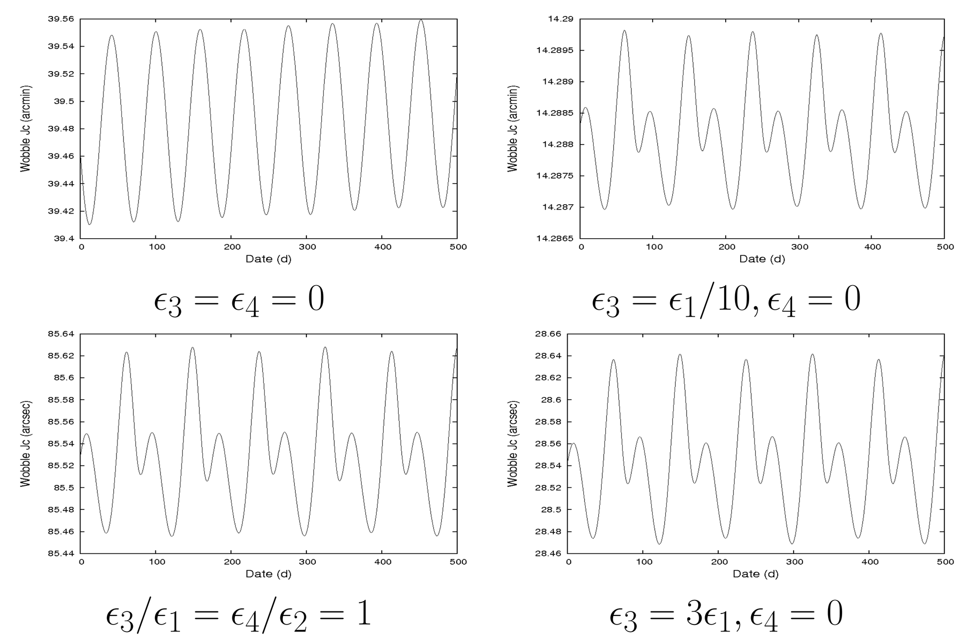

Figure 5: Wobble of the core J_c. We can see that for a spherical core ($\epsilon_3=\epsilon_4=0$) the visual aspect is very different from the other cases. Moreover, a long-term visualisation of J_c shows a slope, i.e. a secular increase of the wobble of the core, while the other cases (out of the resonance) do not show it.

The polar motion of Mercury is very small and do not exhibit significant variations due to the shape of the core. On the contrary, the wobble of the core has an amplitude decreasing when the polar flattening ϵ_3 increases. The obliquity has 2 possible behaviors: when the core is spherical, and when it is not. From a mathematical point of view, what can be seen is whether the system is resonant (resonance between the proper frequency of the core $1/T_z$ and the spin frequency) or not.

Conclusion

This problem is of high interest from a mathematical point of view since it induces a resonance involving the core. From a physical point of view, we show that if the Poincaré model is valid, then the shape of the core cannot be detected from observations of the rotation of Mercury. The polar flattening of the core has an impact on the motion of the fluid, that could be indirectly detected (through the magnetic field?).

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