Modeling the obliquity of Mercury

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Measuring the obliquity of Mercury is crucial to determine its internal structure, in particular its polar inertial momentum C. The well-known Peale's experiment [8] tells us that this quantity, combined with the measurements of the longitudinal librations, gives the size of Mercury's molten core. The inversion of the obliquity is classically made thanks to Peale's formula. This formula considers the gravity coefficients J_2 and C_{22}, and assumes the inclination of Mercury and its node velocity to be constant with respect to a Laplace Plane that must be carefully determined.

We here propose an alternative formula, that is Laplace-Plane free, and considers additional coefficients like J_4, and secular variations of Mercury's dynamics. We are in good agreement with Peale's formula. This allows us to estimate the accuracy of the theoretical modelization of Mercury's obliquity, and to quantify the influence of usually neglected effects.

The problem

The obliquity of Mercury is usually inverted with Peale's formula [7]:

$$\epsilon = -\frac{c\dot{\Omega}\sin\iota}{c\dot{\Omega}\cos\iota + 2n\left(\frac{7}{2}e - \frac{123}{16}e^3\right)C_{22} - n\left(1-e^2\right)^{-3/2}C_{20}}$$

Problems:

- The orbital elements ι, Ω, e are not constant (not enough accurate for inversion of s/c measurements)
- Defined with respect to a Laplace Plane whose definition is not robust
- Other harmonics are now known (Tab.1)

Table 1: The gravity field of Mercury (Smith et al. 2012 [9]).

$C_{20} = -J_2$	$(-5.031 \pm 0.02) \times 10^{-5}$
C_{22}	$(8.088 \pm 0.065) \times 10^{-6}$
$C_{30} = -J_3$	$(-1.188 \pm 0.08) \times 10^{-5}$
$C_{40} = -J_4$	$(-1.95 \pm 0.24) \times 10^{-5}$

So we propose to revisit the determination of this obliquity in trying to consider new effects, like time, tides and the gravity field of Mercury.

A new analytical formula

To validate our numerical approach we rederive a variant of Peale's formula but including the higher order harmonics C_{30} and C_{40}. In this simplified approach we:

1. neglect the planetary perturbations,
2. neglect the influence of the rotation on the orbit,
3. neglect the polar motion of Mercury,
4. express the resonant argument (Mercury is in 3:2 spin-orbit resonance),
5. average over the short periods of the system,
6. determine the dynamical equilibrium,

and we get the following formula, consistent with Peale's:

$$\epsilon = \left(1 + \frac{2}{3}\frac{\dot{\Omega}}{n}\cos(i)\right) \times \frac{c\dot{\Omega}/n\sin(i)}{2C_{22} \times \frac{7}{2}e - C_{20}\left(1+\frac{3}{2}e^2\right) + C_{40}\left(\frac{R}{a}\right)^2\left(\frac{5}{2}+\frac{25}{2}e^2\right) - \frac{2}{3}\left(\frac{\dot{\Omega}}{n}\right)^2 c\sin(i)^2}.$$

A numerical formula

The idea is here to consider the secular *variations of the orbital elements* in a numerical model. For that, we express the orbital elements as sums of trigonometric series, obtained after fit on real ephemerides.

A numerical fit of long-term simulations of the rotation of Mercury allows us to write

$$K(t) = I(t) + \sum_{i=1}^{16} a_i\cos(\omega_i t + \phi_i),$$

$$\sigma_3(t) = \sum_{i=17}^{34} a_i \sin(\omega_i t + \phi_i),$$

with

$$a_i = \frac{c}{\alpha c + \beta C_{20} + \gamma C_{22} + \delta},$$

where K is the obliquity with respect to the ecliptic, I the inclination, and σ_3 the resonant argument associated with the precessional motion, often assumed to be null. From these quantities the instantaneous obliquity $\epsilon(t)$ can be straightforwardly derived.

Results

The Fig.1 represents the time evolution of the obliquity of Mercury, we can see free librations that are expected to be damped, and shorter period variations. Moreover, we see a slope that Peale's formula does not predict.

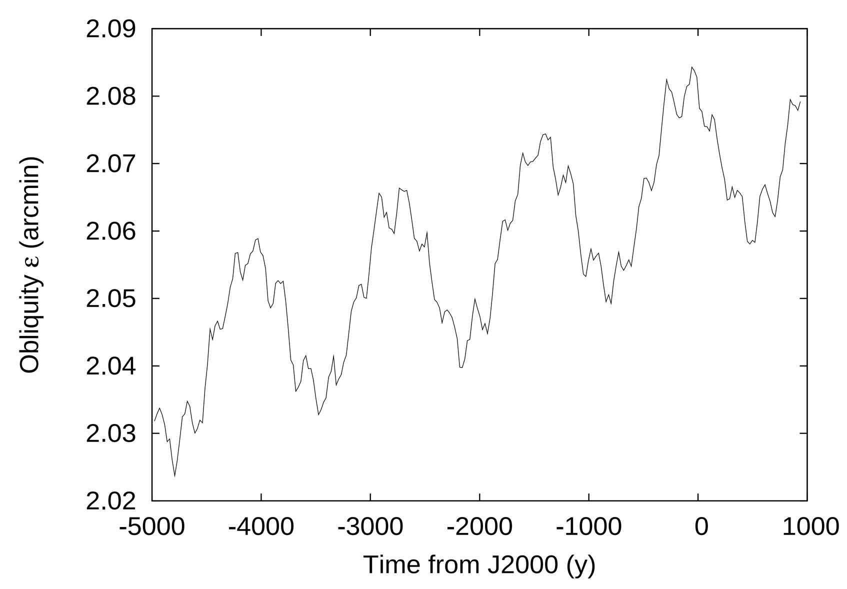

Figure 1: Long-term evolution of Mercury's obliquity.

We also use our formula to predict the influence of tides, using (see e.g. [10])

$$C_{20}(t) = C_{20}^{static} - \frac{3}{2}k_2\frac{M}{m}\left(\frac{R}{a}\right)^3 e\cos\mathcal{M},$$

$$C_{22}(t) = C_{22}^{static} + \frac{3}{4}k_2\frac{M}{m}\left(\frac{R}{a}\right)^3 e\cos\mathcal{M},$$

$$C(t) = C^{static} - \frac{3}{2}k_2\frac{M}{m}\left(\frac{R}{a}\right)^3 emR^2\cos\mathcal{M},$$

and we get short-period variations of ≈ 100 mas (Fig.2).

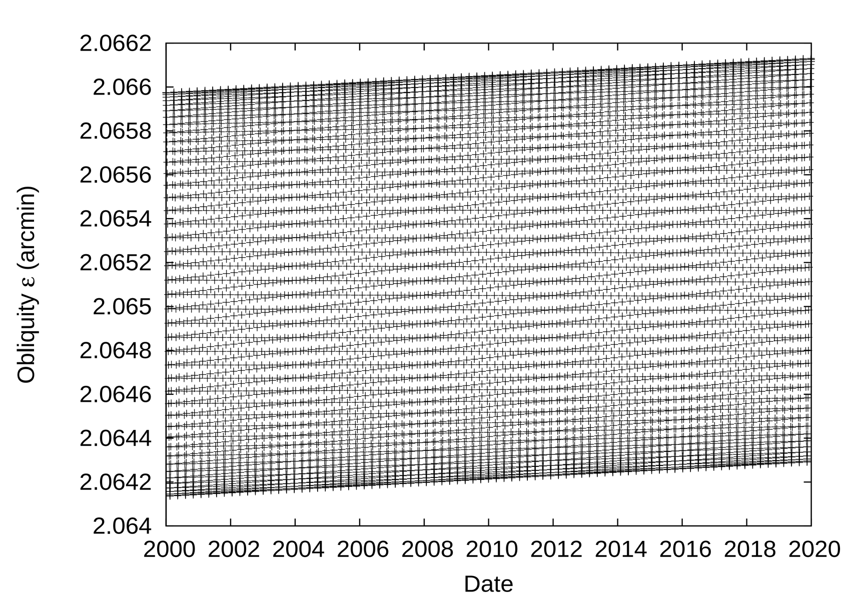

Figure 2: Short-term evolution of the obliquity, in considering tides. The slope is due to the secular variations of the orbital elements.

Using our initial conditions obtained with our numerical formulae to integrate the full equations of the problem shows a small dependency on $J_3 = -C_{30}$ (Fig.3).

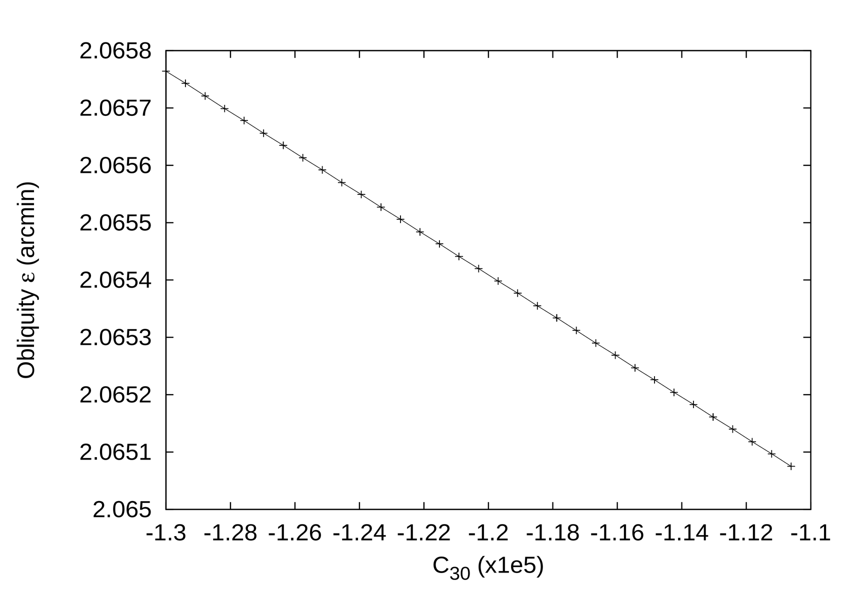

Figure 3: Influence of J_3.

In our study we investigated the influence of different effects on the obliquity of Mercury, i.e. their influence on the determination of the polar moment of inertia C. We summarize the results in Tab.2.

Table 2: Influence on the mean obliquity of usually neglected effects.

C_{30}	≈ 250 mas
Tides	≈ 100 mas
Polar motion	≈ 80 mas [4]
Short-period librations	< 20 mas [1]
Secular drift	≈ 10 mas over 20 years
C_{40}	negligible

Inverting the observed obliquity

From Earth-based radar observations of the spin-pole direction of Mercury (Margot et al. [2, 3]) we see that the planet is actually at the Cassini state 1, and that $\epsilon = 2.04 \pm 0.08$ arcmin. Using Peale's formula, Margot et al. [3] get $c = 0.346 \pm 0.014$.

Table 3: Inverting Margot's obliquity.

Formula	$c = C/(mR^2)$
Peale's	0.346 ± 0.014
Analytical	0.34621 ± 0.01358
Numerical	0.34576 ± 0.01349
Analytical $+J_3$	0.34550 ± 0.01357
Numerical $+J_3$	0.34506 ± 0.01348

We converge!

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Further reading

This study is extensively detailed in (Noyelles & Lhotka 2013 [6]).

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